Lifetime-Optimal Data Routing in Wireless Sensor Networks Without Flow Splitting

Y. Thomas Hou[*] Yi Shi
Virginia Tech
The Bradley Dept. of Electrical
and Computer Engineering
Blacksburg, VA, USA
{thou,yshi}@vt.edu

Jianping Pan
University of Waterloo
Dept. of Electrical and
Computer Engineering
Waterloo, Ontario, Canada
jpan@bbcr.uwaterloo.ca

Scott F. Midkiff
Virginia Tech
The Bradley Dept. of Electrical
and Computer Engineering
Blacksburg, VA, USA
midkiff@vt.edu

Abstract

We consider two-tiered wireless sensor networks, and address the network lifetime problem for upper-tier aggregation and forwarding nodes *(AFNs). Existing flow routing solutions proposed for maximizing network lifetime require AFNs to transmit flows to different nodes at the same time, which we call* multi-session *flow routing solutions. If an AFN is equipped with a single transmitter/receiver pair, a multi-session flow routing solution requires a packet-level power control at the AFN. In this paper, we show that it is possible to achieve the same optimal network lifetime by power control on a much larger timescale with the so-called* single-session *flow routing solutions. More importantly, we show how to perform optimal single-session flow routing when the bit-rate of composite flows generated by AFNs is time-varying, as long as the average bit-rate can be estimated. These results offer new understanding on energy-constrained flow routing in wireless sensor networks.*

1 Introduction

We consider two-tiered wireless sensor networks that can be deployed for high bit rate video sensing applications. This type of sensor networks consists of a number of *sensor clusters* and a *base-station*. Each cluster is deployed around a strategic location, and consists of a number of wireless *micro-sensor nodes* (MSNs) and one *aggregation and forwarding node* (AFN). Each MSN can capture and transmit video data to an AFN that performs in-network information processing by aggregating all correlated data received in the same cluster (*data fusion*). The AFN then sends the composite video data flow to the base-station through single or multi-hop transmission.

One of the most important performance measures for wireless sensor networks is *network lifetime.* For two-tiered wireless sensor networks, whenever an AFN runs out of energy, the video sensing capability for that cluster is completely lost. Therefore, the definition of network lifetime would be the time until any AFN fails due to depletion of energy. Since the lifetime of each individual AFN heavily depends on its energy consumption behavior, and the majority of power consumption at an AFN is due to its radio communication, it is essential to devise strategies that can minimize radio-related power consumption at AFNs. A straight forward approach to reducing energy consumption at an AFN is to reduce the bit rate generated at an AFN via aggregation or compression. But for high resolution video sensing applications, the minimum bit rate requirement at each AFN may still be quite high. Although promising approach to maximizing network lifetime is to control the output power level of radio transmitters. Since the output power level of a radio transmitter directly affects its coverage, it is important to utilize the relay capability among AFNs to forward aggregate flows. This offers an opportunity to dynamically control the output power level of AFNs, so that different network routing topologies can be formed, and network lifetime can be extended.

This paper investigates optimal flow routing among upper-tier AFNs with dynamic power control at AFNs, so that network lifetime can be maximized. Existing solutions to this problem, obtained under linear programming (LP) (see, *e.g.*, [4]), require each AFN to split outgoing data flow into multiple subflows destined to different nodes at the same time, which we call *multi-session flow routing solutions*. With this approach, when an AFN is equipped with a single transmitter/receiver pair, it is necessary for the AFN to perform *packet-level* power control, which is costly to

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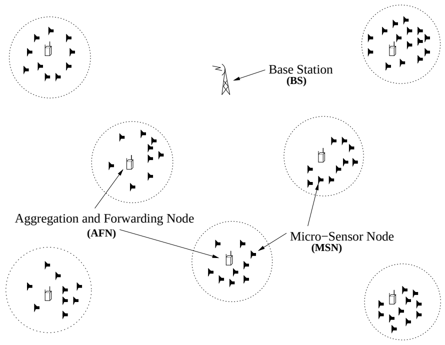
[*] This research has been supported in part by the National Science Foundation under Grants ANI-0312655 and CNS-0347390, and Office of Naval Research under N00014-03-1-0521.

implement in practice, particularly at high bit rate. A naive alternative is to have each AFN be equipped with multiple transmitters, each of them corresponding to an outgoing flow. Since the number of concurrent flows from an AFN is $O(N)$, where N is the number of total AFNs, this approach is clearly not scalable. In this paper, we explore a completely different approach with the so-called *single-session* flow routing solutions where no flow splitting is allowed. We are interested in achieving the same optimal network lifetime by having each AFN perform power control and topology change on a much larger time scale than per-packet level.

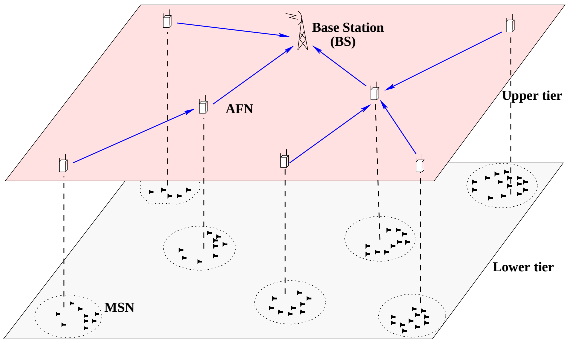
There are several reasons why we are interested in investigating single-session flow routing. First, single-session solutions impose minimum requirement on the power control capability at each AFN (*i.e.*, in a much larger timescale instead of on the per-packet basis). This not only reduces the physical cost of each AFN, but also simplifies control plane operations for the entire network, particularly at for high bit rate sensing applications. Second and perhaps more importantly, the single-session flow routing solution developed in this paper suits perfectly well when directional antennas are employed by AFNs. Directional antennas have significant advantages over omni-directional antennas in terms of minimizing communication interference and reducing power consumption. In this paper, we the lay the theoretical foundation that under an omni-directional antennas, a single-session flow routing solution can achieve the same maximum network lifetime as that with a multi-session flow routing solution. Consequently, this result implies that under directional antennas (where single-session flow routing solution is mandatory in many cases), many folds of network lifetime improvement can be achieved.

In this paper, we first show that an optimal multi-session solution obtained through the LP approach (*e.g.*, [4]) can be transformed into an *equivalent* single-session flow routing solution. By *equivalent*, we mean that the maximum network lifetimes under both approaches are identical. Furthermore, the consumed energy at each AFN must be identical at the end of network lifetime under both approaches. In the second part of this paper, we move on to investigate single-session flow routing solutions when the bit-rate from each AFN is time-varying. We present an equivalence theorem that shows that an optimal single-session flow routing solution for a sensor network of variable bit-rate AFNs can be obtained from an *auxiliary* network of constant bit-rate AFNs. We also show that as long as the estimated average bit-rate is close to the actual value, the network lifetime achieved by single-session flow routing solutions is indeed approaching to the optimum.

The remainder of this paper is organized as follows. In Section 2, we present a reference model for two-tiered wireless sensor networks, and discuss power consumption behavior of upper-tier AFNs. In Section 3, we show how an optimal multi-session flow routing solution can be transformed into an equivalent single-session flow routing solution. Section 4 studies the optimal single-session flow routing problem when the bit-rate from each AFN is time-varying. Section 5 reviews related work, and Section 6 concludes this paper.



(a) Physical topology



(b) A logical topology

Figure 1. Reference architecture for a two-tiered wireless sensor network.

2 Network Reference Model

A Two-tiered Architecture. We focus on a two-tiered architecture for wireless sensor networks, which was motivated by recent advances in distributed source coding (DSC) for sensor networks [5, 16]. Figures 1(a) and (b) show the *physical* topology and a snapshot of the *logical* routing topology of such network, respectively. As shown in these figures, we have three types of nodes in the network: *micro-sensor nodes* (MSNs), *aggregation and forwarding nodes* (AFNs), and a *base-station* (BS). MSNs constitute the *lower-tier* of the network, and are deployed in groups (or clusters) around strategic locations for various sensing applications. Each MSN is small and low-cost, and can be densely deployed within a small geographical area. The objective of an MSN is very simple: once triggered by an event, the MSN starts to capture live data (e.g., video, audio) which it sends directly to the local AFN in one hop. It is worth pointing out that multi-hop routing among MSNs is not necessary due to the small distance between an MSN and the local AFN. By deploying these inexpensive MSNs densely in clusters, and within proximity of a strategic location, it is possible to obtain a comprehensive view of the area by exploring the correlation among the data collected by each MSN.

Within each cluster of MSNs, there is one AFN, which is different from an MSN in terms of its physical structure and logical functions. The primary functions of an AFN include: 1) *data aggregation* (or *fusion*) for data received from the local MSNs, and 2) *forwarding* (or *relaying*) the aggregated composite flows (including flows from other AFNs) to the next-hop AFN toward the base-station. For data fusion, the AFN analyzes the content of each data stream received from MSNs, and then aggregates all the information through DSC [5, 16].

In addition to receiving data streams from MSNs within the local cluster and performing information fusion among the received data, an AFN has an important networking function for the upper-tier AFNs: it serves as a *relay node* for other AFNs to forward their data toward the base-station. Although an AFN is expected to be provisioned with much more energy than an MSN, it also consumes energy at a substantially higher rate (due to wireless communication over greater distances). Consequently, an AFN has a limited lifetime. Upon the depletion of energy at an AFN, the *coverage* for that particular area is lost.

The last component within the two-tiered architecture is the base-station, which is the *sink* node for flows generated by all AFNs in the network. We assume that the base-station has sufficient energy provisioning (e.g., direct power supply), or its energy may be re-provisioned over time. Therefore, the base-station is not subject to the energy constraint considered in this paper.

In summary, the main function of the lower-tier MSNs is data acquisition, while the upper-tier AFNs are used for data fusion and forwarding the aggregated flows toward the base-station. Although the AFNs and base-station are immobile, there is a great degree of flexibility in terms of how the network routing topology can be formed to forward data flows.

Power Consumption Model. A detailed power consumption model for each component in a wireless sensor node can be found in [9]. For an AFN, the radio-related power consumption (*i.e.,* in transmitter and receiver) is the dominant factor [1]. When AFN i transmits data to AFN k with rate f_{ik} b/s, the power consumption at the transmitter can be modeled as

$$p^t_{ik} = c_{ik} \cdot f_{ik} \ . \tag{1}$$

Here, c_{ik} is the power consumption cost of link (i, k), and

$$c_{ik} = \alpha + \beta \cdot d^n_{ik} \ , \tag{2}$$

where α is a *distance-independent* term, β is a coefficient associated with the *distance-dependent* term, d_{ik} is the distance between these two nodes, n is the path loss exponent, and $2 \leq n \leq 4$ [17]. Typical values of these parameters are $\alpha = 50$ nJ/b and $\beta = 0.0013$ pJ/b/m^4 when $n = 4$ [9]. In this paper, we adopt $n = 4$ for all of our numerical results.

The power consumption at the receiver of AFN j can be modeled as [17]:

$$p^r_j = \rho \cdot \sum_{k \neq j} f_{kj} \ , \tag{3}$$

where f_{kj} (also in b/s) is the incoming bit-rate of the composite flow received by AFN j from AFN k. Typical value of ρ is 50 nJ/b [9].

3 Optimal Single-Session Flow Routing

In this section, we show that a multi-session flow routing solution (with flow splitting at AFN) can be transformed into an equivalent single-session flow routing solution (without flow splitting).

3.1 Optimal Multi-Session Flow Routing

Suppose that the data flow's bit-rate generated by AFN i is g_i, and the initial energy at AFN i is e_i. Denote T the network lifetime, *i.e.,* the time duration from network initialization until any AFN drains out of energy. We then have the following incoming/outgoing flow balance equations and energy constraints for each AFN i ($i = 1,\ 2,\ \cdots, N$),

$$g_i + \textstyle\sum_{m \neq i} f_{mi} = \sum_{k \neq i} f_{ik} + f_{iB} \ , \tag{4}$$

$$\rho \textstyle\sum_{m \neq i} f_{mi} T + \sum_{k \neq i} c_{ik} f_{ik} T + c_{iB} f_{iB} T \leq e_i \ , \tag{5}$$

where f_{ik} and f_{iB} denote the flow rate from AFN i to AFN k and to base-station B, respectively. The first set of N equations in (4) states that, at each AFN i, the bit-rate g_i (generated at node i), plus the total bit-rate of incoming flows from other AFNs, is equal to the total bit-rate of outgoing flows. The second set of N inequalities in (5) states that the energy required to receive and transmit all these flows at each AFN i, at the end of network lifetime T, cannot exceed its energy constraint. Our objective is to maximize T while both (4) and (5) are satisfied.

To formulate an optimization problem for network flow routing, let $V_{ik} = f_{ik} T$ and $V_{iB} = f_{iB} T$, where V_{ik} and V_{iB} are the bit-volumes being sent from AFN i to k and B, respectively. We obtain the following linear programming (LP) formulation.

Max $\quad T$
s.t.

$$g_iT+\sum_{m\neq i}V_{mi}-\sum_{k\neq i}V_{ik}-V_{iB}=0 \quad (1\leq i\leq N) \quad (6)$$

$$\sum_{m\neq i}\rho V_{mi}+\sum_{k\neq i}c_{ik}V_{ik}+c_{iB}V_{iB}\leq e_i \quad (1\leq i\leq N) \quad (7)$$

where Eqs. (6) are from the balance equations in (4), and Eqs. (7) are from the energy constraints in (5). Note that T, V_{mi}, V_{ik}, and V_{iB} are variables, and that g_i, ρ, c_{ik}, c_{iB}, and e_i are all constants.

We now have a standard LP formulation, *i.e.*, **Max** cx, **s.t.** $Ax \leq b$ and $x \geq 0$. To reduce variable space and thus computational complexity, we can perform the following pre-processing before running a full-scale LP. For each AFN i, we denote set Q_i containing all the AFNs k satisfying $d_{ik} < d_{iB}$, *i.e.*, AFNs in Q_i are within the radius from AFN i to the base-station B. It is obvious that for AFN i, only AFNs in Q_i may be chosen as relay nodes; that is, we can remove variable f_{ik} when $k \notin Q_i$.

Clearly, such an LP approach will yield a *multi-session* flow routing solution, which has been studied in prior efforts (*e.g.*, see [4]). Under a multi-session flow routing solution, flow splitting is allowed and each AFN may send multiple flows to different nodes at the same time. When an AFN is equipped with a single transmitter/receiver pair, the AFN is required to perform a packet level power control so as to reach different next-hop nodes. In the next subsection, we will explore a completely different approach, which yield *single-session* flow routing solutions where power control and topology change are only done on a much larger time scale instead of on the per-packet basis.

3.2 Transformation to Single-Session Solution

We show that a multi-session flow routing solution can be transformed into an equivalent single-session flow routing solution. By *equivalent*, we mean that both flow routing solutions have the same network lifetime. Besides preserving their flow balance, we also require that the per-node energy consumption at the end of network lifetime are identical under both solutions.

Theorem 1 *Given a multi-session flow routing solution ψ with maximum network lifetime T, there exists an equivalent single-session flow routing solution $\hat{\psi}$ with the same maximum network lifetime T.*

Theorem 1 can be proved by constructing a single-session flow routing solution (denoted as $\hat{\psi}$) for a given multi-session flow routing solution ψ, and showing that $\hat{\psi}$ is equivalent to ψ according to our criteria. Before we perform the transformation, it is important to remove all forward cycles in ψ. This is necessary to ensure that upon the termination of our algorithm, the flow routing of each AFN will be in single-session mode. Here, a flow cycle in ψ refers to a directed cycle composed of directed links each carrying a positive flow. Cycle detection and removal procedures can use depth-first search and mark algorithms, which are discussed in the literature (see, *e.g.*, [6]). Therefore, we will not discuss them further in this paper. It is worth pointing out that after a cycle detection and removal procedure, the network lifetime will be identical to that obtained by solving the LP formulation.

After performing cycle detection and removal procedures, we obtain a cycle-free multi-session flow routing solution ψ with maximum network lifetime T. We are now ready to perform multi-session to single-session transformation. The transformation algorithm follows an *exterior-to-interior* order, *i.e.*, we begin with non-relay AFNs first, and perform the transformation gradually on relay AFNs toward the base-station. This procedure will ensure that, by the time we perform transformation for AFN s, all the AFNs from which AFN s receives flows have already been transformed into single-session mode, and that all incoming flows to AFN s are already determined by earlier transformations on other AFNs.

The key idea of transformation is as follows. For each AFN s, its relay nodes under a single-session flow routing solution will be the same set of relay nodes under the given multi-session solution. However, for single-session solution, we partition network lifetime T into several durations. For each duration segment, AFN s will solely transmit its data to one particular relay node. The length of these time durations during which AFN s will transmit its outgoing flow exclusively to this respective relay node can be determined by the total bit-volume sent to this node under the multi-session flow routing solution.

Under $\hat{\psi}$, denote $\hat{f}_{ik}(t)$ and $\hat{f}_{iB}(t)$ the bit-rates at time t ($0 \leq t \leq T$) from AFN i to AFN k and the base-station B, respectively. Due to the nature of single-session flow routing, at any time $t \in [0, T]$, there is only one flow in the set of $\hat{f}_{ik}(t)$ and $\hat{f}_{iB}(t)$ that has a non-zero bit-rate.

Algorithm 1 *(***Multi-Session to Single-Session Transformation***) For a cycle-free multi-session flow routing solution ψ with maximum network lifetime T, the following iterative algorithm obtains an equivalent single-session flow routing solution $\hat{\psi}$.*

1. *Identify a multi-session AFN s such that*
   - (a) *either s is not receiving flows from any other AFN (*i.e., *a non-relay AFN), or*
   - (b) *all AFNs from which AFN s receives flows are already in single-session mode.*

Table 1. AFN coordinates, local flow rate, and initial energy of the example sensor network

AFN i	$(x_i,\ y_i)$ (m)	g_i (kb/s)	e_i (kJ)
1	$(150,\ 20)$	9	28
2	$(50,\ 150)$	7	26
3	$(150,\ 40)$	5	38
4	$(110,\ 80)$	1	19
5	$(110,\ 120)$	3	21

If there does not exist such a multi-session AFN, we already have an equivalent single-session flow routing solution $\hat{\psi}$; otherwise, perform the following transformation for AFN s.

2. *For AFN s, denote $R_s = r_1, r_2, \cdots, r_{|R_S|}$ the set of relay nodes for AFN s under multi-session solution ψ. If s has a direct flow to the base-station B under ψ, B is also included in R_s. Let $|R_s|$ denote the number of nodes in R_s. We define $|R_s|$ time duration segments for the single-session solution,* i.e., *$T_{s,r_1} = [0, t_1)$, $T_{s,r_2} = [t_1, t_2)$, $\cdots$, $T_{s,r_{|R_s|}} = [t_{|R_s|-1}, t_{|R_s|}]$. T_{s,r_k} $(k = 1, 2, \cdots, |R_s|)$ are defined as follows:*

$$\int_{T_{s,r_k}} \left[g_s + \sum_{m\neq s} \hat{f}_{ms}(t) \right] dt = f_{s,r_k} T \ . \qquad (8)$$

We will show $t_{|R_s|} = T$ in the correctness proof for this algorithm. Then, we have a single-session flow routing schedule for AFN s as follows:

$$\hat{f}_{s,r_k}(t) = \begin{cases} g_s + \sum_{m\neq s} \hat{f}_{ms}(t) & t \in T_{s,r_k} \ , \\ 0 & \textit{otherwise,} \end{cases} \qquad (9)$$

i.e., during T_{s,r_k}, AFN s will solely transmit to node r_k, where $k = 1,\ 2,\ \cdots,\ |R_s|$.

3. *Go to Step* 1.

To show that Algorithm 1 is correct, it is sufficient to show that the following two criteria are satisfied: 1) For each AFN, the rate of incoming (including self-generated) flows is equal to the rate of outgoing flow (*i.e.*, flow balance) at any time, and 2) at time T, the energy consumption at each AFN under $\hat{\psi}$ is the same as that under ψ. A complete proof is available in [10] and is omitted here to conserve paper length.

3.3 A Numerical Example

We use a 5-AFN network to illustrate how to transform a multi-session flow routing solution into an equivalent single-session flow routing solution by Algorithm 1.

Table 2. Inter-node flow rates in a multi-session solution for Example 1

i	f_{ik} (kb/s)					f_{iB} (kb/s)
	$k=1$	$k=2$	$k=3$	$k=4$	$k=5$	
1	0	0	1.1229	5.4243	2.4528	0
2	0	0	0	0	0	7.0000
3	0	0	0	0	2.4320	3.6909
4	0	0	0	0	0	6.4342
5	0	0	0	0	0	7.8848

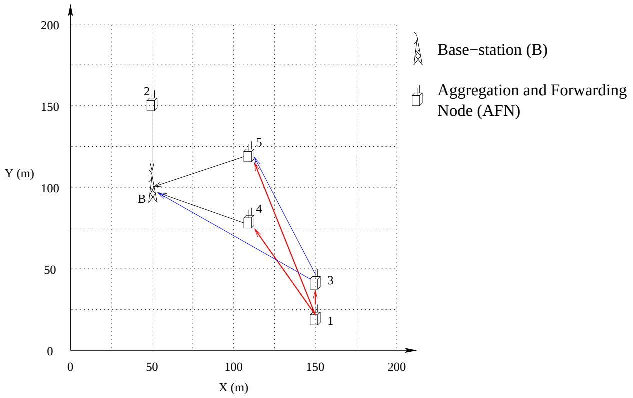
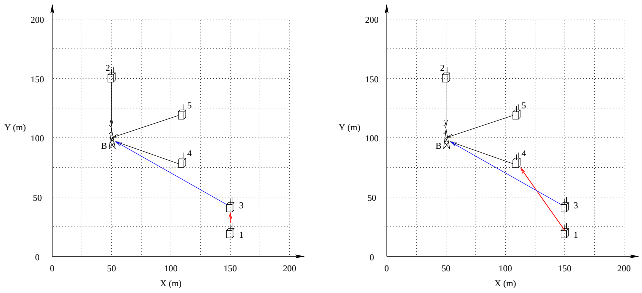


Figure 2. A multi-session flow routing solution for the sample sensor network.

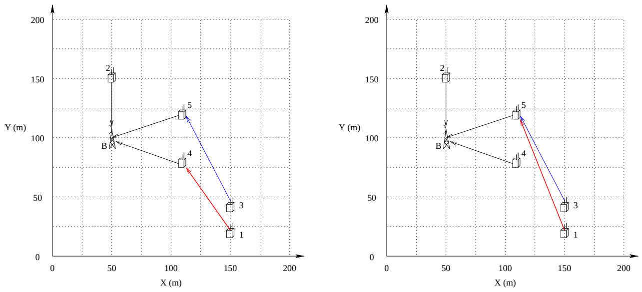
Example 1 *Referring to Fig. 2, suppose that we have* 5 *AFNs. The coordinates $(x_i,\ y_i)$, local flow rate g_i, and initial energy e_i for each AFN i are listed in Table 1. The base-station (B) is located at* $(50, 100)$ *m.*

With the LP approach (see Section 3.1), we obtain a multi-session flow routing solution (see Fig. 2) with f_{ik} and f_{iB} listed in Table 2. For the given initial energy at each AFN, the maximum network lifetime obtained by this multi-session solution is $T = 302.88$ days.

We now use Algorithm 1 to transform the above multi-session flow routing solution into a single-session flow routing solution. According to Algorithm 1, since nodes 2, 4, *and* 5 *are already in single-session mode, there is no need to perform transformation on them (except that the flow rates of* 4 *and* 5 *need to be recomputed). We then transform AFN* 1 *to a single-session routing schedule. That is, since $\int_{T_{13}} g_1 dt = f_{13}T$ and only T_{13} is unknown, we obtain $T_{13} = [0,\ 37.79)$ (in days). Similarly, we have $T_{14} = [37.79,\ 220.33)$ and $T_{15} = [37.79,\ 220.33]$. That is, during $[0,\ 37.79)$ days, AFN* 1 *sends its outgoing flow to AFN* 3*; during $[37.79,\ 220.33)$ days, AFN* 1 *sends its outgoing flow to AFN* 4*; during $[220.33, 220.33]$ days, AFN* 1 *sends its flow to AFN* 5*. Following Algorithm 1, we proceed to transform AFN* 3 *as follows: during $[0,\ 155.56)$ days, AFN* 3 *sends all its flow to base-station B; during $[155.56,\ 302.88]$ days, AFN* 3 *sends all its flow to AFN* 4.



(a) $[0,\ 37.79)$ days (b) $[37.79,\ 155.56)$ days



(c) $[155.56,\ 220.33)$ days (d) $[220.33,\ 302.88]$

Figure 3. An equivalent single-session flow routing schedule during $[0,\ 302.88]$ days for Example 1.

Figure 3 shows the entire single-session flow routing schedule during network lifetime of 302.88 *days. It is easy to verify that the flow balance equation at each AFN is satisfied throughout* $[0,\ 302.88]$ *days, and that at the end of* 302.88 *days, the energy consumption at each AFN is the same as that under the multi-session flow routing solution.*

3.4 Discussions

It is important to note that the single-session flow routing solution developed in this paper is fundamentally different from a TDM-based scheme. First and foremost, under a TDM-based scheme, there is a *regular* time-frame that each sender shall follow to send information in a specific time-slot within the frame *periodically*. Under single-session flow routing, an AFN can send flows to one node only within a specific time duration, and will no longer send to this node again at any other time. Second, the time scale of a TDM-based scheme is typically small with deterministic patterns. Under single-session flow routing, the time scale to change next hop node is much larger (see example in the last subsection). Finally, our single-session flow routing solution meets the stringent requirement of satisfying flow balance and more important, the energy constraint at AFNs, which may not be the focus under a TDM-based scheme.

4 Extension to Variable Bit-Rate

In this section, we relax the constant bit-rate constraint for g_i at each AFN i. We show that as long as the *average* bit-rate (denoted as $\bar{g}_i$) for $g_i(t)$ can be estimated, the optimal single-session flow routing solution is also obtainable. As an example, if the bit rate from an AFN follows an on/off process with known average bit-rate, we show how to obtain an optimal single-session flow routing solution to maximize network lifetime. In addition, we show that as long as the estimated bit-rate $\bar{g}_i$ does not deviate too much from the actual value, the network lifetime obtained through single-session flow routing is near-optimal.

4.1 Perfect Knowledge of Average Bit-Rate

We begin with the ideal case that we have perfect knowledge of the average bit-rate of the flow generated by AFN i, denoted as $\bar{g}_i$. In this subsection, we show that an optimal single-session flow routing solution for a sensor network of variable bit-rate AFNs can be obtained by studying the optimal single-session flow routing solution for an auxiliary network of constant bit-rate AFNs.

Denote P as the problem of variable bit-rate AFNs. The initial energy at AFN i is e_i, and each AFN generates a flow at rate $g_i(t)$. Denote $\bar{P}$ as the problem of constant bit-rate AFNs with the same network configuration and initial energy at each AFN. Under $\bar{P}$, each AFN is assumed to generate a constant bit-rate composite flow with rate $\bar{g}_i$, which is the estimated average of $g_i(t)$, *i.e.*

$$\bar{g}_i = E[g_i(t)]\ . \tag{10}$$

The following theorem shows that for a flow solution for $\bar{P}$ with maximum network lifetime T, there exists an equivalent solution for P with the same network lifetime T.

Theorem 2 *For a constant bit-rate problem $\bar{P}$ with maximum network lifetime T and the corresponding optimal flow routing solution $\bar{\pi}$, there exists an equivalent single-session flow routing solution π for the equivalent variable bit-rate problem P with the same network lifetime T.*

Theorem 2 can be proved by constructing a single-session flow routing solution for P with the same network lifetime as that obtained for $\bar{P}$. In the following algorithm, we show how to construct such a single-session flow routing solution. Not surprisingly, this algorithm follows closely to Algorithm 1, with the difference being that g_i is now replaced by $g_i(t)$. Again, we need to first perform the cycle detection and removal procedure to ensure that the multi-session flow routing solution $\bar{\pi}$ for $\bar{P}$ is cycle-free before the transformation.

Algorithm 2 *Given a flow routing solution $\bar{\pi}$ for constant bit-rate problem $\bar{P}$ with maximum network lifetime T, the following iterative operations provide an equivalent single-session flow routing solution π for variable bit-rate problem P with the same network lifetime T.*

Denote $\bar{f}_{ik}$ and $\bar{f}_{iB}$ the flow rates from AFN i to AFN k and to base-station B under $\bar{\pi}$, $f_{ik}(t)$ and $f_{iB}(t)$ the flow rates from AFN i to AFN k and to base-station B at time t under π, respectively.

1. *Under $\bar{\pi}$, identify a multi-session AFN s such that*
   - (a) *either s is not receiving flows from any other AFN (*i.e.*, a non-relay AFN), or*
   - (b) *the incoming flows for AFN s in P are already defined.*

   If no such AFN exists, we already have an equivalent single-session flow routing solution π for P; otherwise, define the following outgoing flows for s in P.

2. *For AFN s, denote $R_s = r_1, r_2, \cdots, r_{|R_S|}$ be the set of relay nodes of s in $\bar{P}$ (the base-station is also included if s sends flow to B under $\bar{\pi}$). Here, $|R_s|$ denotes the number of AFNs in R_s. Define $|R_s|$ durations, $T_{s,r_1} = [0, t_1)$, $T_{s,r_2} = [t_1, t_2)$, $\cdots$, $T_{s,r_{|R_s|}} = [t_{|R_s|-1}, t_{|R_s|}]$. Again, it can be shown that $t_{|R_s|} = T$. T_{s,r_k} ($k = 1, 2, \cdots, |R_s|$) are defined as follows:*

   $$\int_{T_{s,r_k}} \left[g_s(t) + \sum_{m \neq s} f_{ms}(t) \right] dt = \bar{f}_{s,r_k} T \ . \quad (11)$$

   During T_{s,r_k}, AFN s will only transmit to AFN r_k. Then, the single-session flow routing schedule at AFN s for P is

   $$f_{s,r_k}(t) = \begin{cases} g_s(t) + \sum_{m \neq s} f_{ms}(t) & t \in T_{s,r_k} \ , \\ 0 & \textit{otherwise.} \end{cases} \quad (12)$$

3. *Go to Step 1.*

The correctness proof for Algorithm 2 follows the same token as the correctness proof for Algorithm 1, and is thus omitted it here to conserve paper length. There is one detail that we should pay special attention. In the correctness proof for Algorithm 2, we assume that

$$\bar{g}_s = \frac{1}{T} \int_0^T g_s(t) dt \ ,$$

which means that the estimated bit-rate $\bar{g}_s$ is the actual average bit-rate over time interval T. In practice, $\bar{g}_s$ may deviate slightly from $\frac{1}{T}\int_0^T g_s(t)dt$, which we will discuss in the next subsection.

Theorem 2 and Algorithm 2 show that for problem P, we can obtain a single session flow routing solution π with the same network lifetime T, where T is the maximum network lifetime that is achievable for problem $\bar{P}$ with multi-session flow routing solution $\bar{\pi}$. The next theorem shows that this network lifetime T is also the maximum achievable network lifetime for P. Consequently, the single-session flow routing solution π obtained by Algorithm 2 is also optimal.

Theorem 3 *(*π **is Optimal***) The single-session flow routing solution π obtained by Algorithm 2 is optimal in terms of maximizing network lifetime for problem P.*

Proof. It is sufficient to show that the maximum network lifetime for problem P is the same as the maximum network lifetime for problem $\bar{P}$. First, since Theorem 2 shows that there is a solution for problem P with lifetime T, where T is the maximum network lifetime for problem $\bar{P}$, then the maximum network lifetime for problem P should be greater than or equal to T. We now show that the maximum network lifetime for problem $\bar{P}$ is also greater than or equal to the maximum network lifetime for problem P. With these two results, we conclude that the maximum network lifetime for problem P is the same as the maximum network lifetime for problem $\bar{P}$.

To show that the maximum network lifetime for problem $\bar{P}$ is indeed greater than or equal to the maximum network lifetime for problem P, it is sufficient to prove that, for a network flow routing solution π under P with the maximum network lifetime τ, we can find an equivalent flow routing solution $\bar{\pi}$ under $\bar{P}$ with the same network lifetime τ.

Since π is a network flow routing solution for P, for each AFN i, we have the following flow balance,

$$f_{iB}(t) + \sum_{k \neq i} f_{ik}(t) = g_i(t) + \sum_{m \neq i} f_{mi}(t) \ . \quad (13)$$

We also have the following energy constraint inequality,

$$\int_0^{\tau} \left[\sum_{m \neq i} \rho f_{mi}(t) + \sum_{k \neq i} c_{ik} f_{ik}(t) + c_{iB} f_{iB}(t) \right] dt \leq e_i \ . \quad (14)$$

We now construct a flow routing solution $\bar{\pi}$ for $\bar{P}$ that has the same network lifetime τ. For $\bar{\pi}$, we define

$$\bar{f}_{ik} = \frac{\int_0^{\tau} f_{ik}(t)dt}{\tau} \ , \quad (15)$$

$$\bar{f}_{iB} = \frac{\int_0^{\tau} f_{iB}(t)dt}{\tau} \ . \quad (16)$$

We show that through such a construction, both the flow balance equation and energy constraint are satisfied for $\bar{P}$. Consequently, $\bar{\pi}$ is a feasible flow routing solution for $\bar{P}$.

For flow balance, we have

$$
\begin{aligned}
& \bar{g}_i + \sum_{m\neq i} \bar{f}_{mi} = \frac{1}{\tau}\left[\int_0^\tau g_i(t)dt + \sum_{m\neq i}\int_0^\tau f_{mi}(t)dt\right] \\
= \; & \frac{1}{\tau}\left[\int_0^\tau f_{iB}(t)dt + \sum_{k\neq i}\int_0^\tau f_{ik}(t)dt\right] = \bar{f}_{iB} + \sum_{k\neq i}\bar{f}_{ik} \; .
\end{aligned}
$$

The first equality holds by our assumption that $g_s = \frac{1}{\tau}\int_0^\tau g_s(t)dt$ and by (15). The second equality holds due to the flow balance equation (13). The third equality holds due to (15) and (16).

Similarly, for the energy constraint, we have

$$
\begin{aligned}
& \sum_{k\neq i} \rho \bar{f}_{ki}\tau + \sum_{k\neq i} c_{ik}\bar{f}_{ik}\tau + c_{iB}\bar{f}_{iB}\tau \\
= \; & \int_0^\tau \left[\sum_{k\neq i}\rho f_{ki}(t) + \sum_{k\neq i} c_{ik} f_{ik}(t) + c_{iB} f_{iB}(t)\right] dt \\
\leq \; & e_i \; .
\end{aligned}
$$

The first equality holds due to (15) and (16) and the inequality holds due to (14). Thus, at time τ, the energy consumption at each AFN i under $\bar{\pi}$ for problem $\bar{P}$ is the same as that under π for problem P, *i.e.*, the network lifetime under $\bar{\pi}$ is also τ for $\bar{P}$. Therefore, for the maximum network lifetime τ under P, we can find a flow routing solution under $\bar{P}$ that has the same network lifetime. This completes the proof. □

The significance of Theorem 2 and Theorem 3 is that they enable us to obtain an optimal single-session flow routing solution for a sensor network of variable bit-rate AFNs (*e.g.*, following an on/off process), as long as the estimated average bit-rate of each AFN is the same as its actual value. In a nutshell, this approach takes the following two steps.

- First, we find an optimal multi-session flow routing solution $\bar{\pi}$ for problem $\bar{P}$ (from the LP problem described in Section 3.1).
- Second, we apply Algorithm 2 to get an optimal single-session flow routing solution for problem P.

4.2 Imperfect Estimate of Average Bit-Rate

Our investigation in the last subsection assumes that the estimated average bit-rate $\bar{g}_i$ matches perfectly with the actual value, *i.e.*, $\bar{g}_i = \frac{1}{T}\int_0^T g_i(t)dt$. In practice, the estimated average bit-rate for $g_i(t)$ could deviate from the actual value for $\bar{g}_i(t)$ over network lifetime T. We now show that as long as this discrepancy is not substantial, the procedure developed in the last subsection can still yield a near-optimal single-session flow routing solution. Furthermore, the deviation between the actual network lifetime and the expected maximum network lifetime is negligible, as long as the estimated average bit-rate $\bar{g}_i$ is not far away from the actual value $\frac{1}{T}\int_0^T g_i(t)dt$, where T is the actual network lifetime. We use the following example to illustrate this result, which has the dual purpose of illustrating the procedures to obtain a single-session flow routing solution in Section 4.1.

Table 3. Traffic “on" periods and bit rate during “on” periods for each AFN (k is non-negative integer).

AFN	“on” period (in days)	Rate (kb/s)
1	$[k,\ k+0.4]\bigcup[k+0.8,\ k+1]$	15
2	$[k,\ k+0.3]\bigcup[k+0.6,\ k+1]$	10
3	$[k+0.4,\ k+0.9]$	10
4	$[k+0.2,\ k+0.4]$	5
5	$[k,\ k+0.3]$	7.5

Example 2 We use the sample network configuration in Fig. 2, where there are 5 AFNs and a base-station (B). Each AFN's coordinates and initial energy are the same as those in listed Table 1. The base-station is also located at the same location (*i.e.*, $(50, 100)$ m). The local flow bit-rate g_i listed in Table 1 now represents the estimated average bit-rate $\bar{g}_i$ for AFN i, *i.e.*, $\bar{g}_1 = 9$ kb/s for AFN 1, $\bar{g}_2 = 7$ kb/s for AFN 2, $\bar{g}_3 = 4$ kb/s for AFN 3, $\bar{g}_4 = 1$ kb/s for AFN 4, and $\bar{g}_5 = 3$ kb/s for AFN 5. Assume that $g_i(t)$ (in kb/s) follows a periodic on/off process (see Table 3).

Clearly depending on the actual network lifetime T, the average rate for each AFN i over time T (*i.e.*, $\frac{1}{T}\int_0^T g_i(t)dt$) could be slightly different from its estimated average $\bar{g}_i$. We will show such slight discrepancy results in negligible difference between the actual network lifetime T and the estimated maximum network lifetime (denoted as $\bar{T}$).

Denote the flow routing problem for the network of variable bit-rate AFNs as P and the flow routing problem for the network of constant bit-rate AFNs as $\bar{P}$. Under $\bar{P}$, we assume that each AFN i generates a constant bit-rate flow $\bar{g}_i$, which is the estimated average bit-rate for AFN i. We can build an LP problem (see Section 3.1) to get an optimal multi-session flow routing solution for $\bar{P}$ (see Fig. 2) with exactly the same $\bar{f}_{ik}$ and $\bar{f}_{iB}$ listed in Table 2. Again, the maximum network lifetime for $\bar{P}$ of the sample sensor network is $\bar{T} = 302.88$ days.

Now we move on to obtain a single-session flow routing solution for P. According to Algorithm 2, since AFNs 2, 4, and 5 are already in single-session mode, there is no need to perform transformation on these AFNs. For AFN 1, since it sends flows to AFNs 3, 4, and 5 under $\bar{P}$, we calculate T_{13}, T_{14}, and T_{15} using (11). That is, since $\int_{T_{13}} g_1(t)dt = f_{13}T$ and only T_{13} is unknown, we obtain $T_{13} = [0,\ 37.87)$ (in days). Therefore, during $[0,\ 37.87)$

Table 4. Single-session flow routing schedule for Example 2.

AFN	Time Duration (in days)	Next-Hop Node	Flow Bit-Rate
1	$[0,\ 37.87)$	3	$g_1(t)$
	$[37.87,\ 220.20]$	4	$g_1(t)$
	$[220.20,\ 302.38]$	5	$g_1(t)$
2	$[0,\ 302.38]$	B	$g_2(t)$
3	$[0,\ 37.87)$	B	$g_1(t) + g_3(t)$
	$[37.87,\ 155.68)$	B	$g_3(t)$
	$[155.68,\ 302.38]$	5	$g_3(t)$
4	$[0,\ 37.87)$	B	$g_4(t)$
	$[37.87,\ 220.20)$	B	$g_1(t) + g_4(t)$
	$[220.20,\ 302.38]$	B	$g_4(t)$
5	$[0,\ 155.68)$	B	$g_5(t)$
	$[155.68,\ 220.20)$	B	$g_3(t) + g_5(t)$
	$[220.20,\ 302.38]$	B	$g_1(t) + g_3(t) + g_5(t)$

days, AFN 1 sends its flows to AFN 3. Similarly, we obtain that $T_{14} = [37.87,\ 220.20)$ and $T_{15} = [220.20,\ 302.93]$ (in days) with $\int_{T_{14}} g_1(t)dt = f_{14}T$ and $\int_{T_{15}} g_1(t)dt = f_{15}T$, respectively. That is, AFN 1 sends its flows to AFN 4 during $[37.87,\ 220.20)$ days and sends its flows to AFN 5 during $[220.20,\ 302.93]$ days. Note that the actual lifetime for AFN 1 (302.93 days) is slightly different from the expected network lifetime (302.88 days), due to the imperfect average bit-rate estimation for $g_i(t)$ with $\bar{g}_i$.

For AFN 3, since it sends flows to AFN 5 and base-station B under $\bar{P}$, we calculate T_{35} and T_{3B} under P. Since $\int_{T_{3B}}[g_3(t) + f_{13}(t)]dt = f_{3B}T$, we obtain $T_{3B} = [0,\ 155.68)$ (in days). Similarly, since $\int_{T_{35}}[g_3(t) + f_{13}(t)]dt = f_{35}T$, we obtain $T_{35} = [155.68,\ 302.84]$ (in days). Therefore, AFN 3 sends all its flows to base-station B during time $[0,\ 155.68)$ and sends all its flows to AFN 5 during time $[155.68,\ 302.84]$. Again, we note that the actual lifetime for AFN 3 (302.84 days) is slightly different from the expected network lifetime (302.88 days), due to the same average bit-rate estimation error.

We can easily compute the node lifetimes of AFNs 2, 4, and 5, and find that AFN 4 has the smallest life 302.38. Since AFN 4 has the smallest lifetime among all the AFNs, it is also the network lifetime. Note that this is very close to the maximum network lifetime under $\bar{P}$ (302.88 days). We now have a single-session flow routing solution for P, which is summarized in Table 4. It is easy to verify that the incoming/outgoing flow balance holds for each AFN at any time during $[0,\ 302.38]$, with the bit-rate of composite flows generated by each AFN, $g_i(t)$, defined in Table 3.

We can also verify that there is indeed a tiny deviation here between the estimated average bit-rate $\bar{g}_i$ and the actual average bit-rate for each AFN i during $[0,\ 302.38]$ days. For example, the actual average bit rate of AFN 1 over $[0,\ 302.38]$ is $\frac{1}{302.38}\int_0^{302.38} g_1(t)dt = 9.0075$, which is very close the the estimated average bit-rate for $g_1(t)$, 9. Similarly, the actual average bit-rates for AFNs 2, 3, 4, and 5 over time interval $[0,\ 302.38]$ days are 7.0011, 4.9937, 1.0017, and 3.0062 (all in kb/s), which are very close to the estimated averages bit-rates 7, 5, 1, and 3, respectively. $\square$

5 Related Work

There has been active research on addressing energy conservation issues in wireless sensor networks. In this section, we briefly summarize related research efforts on power control, power-aware routing, and network lifetime maximization.

Power control capability has been studied at different layers in recent years. At the *network* layer, most work on the power control problem can be classified into two categories. The first category is comprised of strategies to find an optimal transmitter power to control the *connectivity* properties of the network (see, *e.g.*, [11, 14, 15, 18, 21]). A common theme in these strategies is to formulate power control as a network layer problem, and then to adjust each node's transmission power, so that a different network connectivity topology can be formed for different objectives. The second category is usually referred to as *power-aware routing*. Most schemes use a shortest path algorithm with a power-based metric, rather than a hop-count based metric (see, *e.g.*, [7, 8, 13, 20]). However, energy-aware (*e.g.*, minimum energy path) routing may not ensure good performance in maximum network lifetime [19].

The notion of network lifetime for wireless sensor networks has been discussed in [3]. The most relevant work on network lifetime related to our research have been described in [4]. Here, we describe some additional relevant work on maximizing network lifetime. In [2], Bhardwaj and Chandrakasan attempted to develop a bound for maximum network lifetime through the notion of *role assignment*, which corresponds to the single-session solution discussed in this paper. But since the transformation from multi-session solution to single-session solution was not explored, their approach resulted in prohibitively complex problem formulation, and polynomial solutions only exist for very simple scenarios. In [12], Kalpakis *et al.* proposed a so-called *GETTREE* algorithm, which can be extended to give a single-session solution. The algorithm was obtained by applying results from graph theory, without exploring some unique properties of these networks (*e.g.*, bit-volume conservation between equivalent solutions). Consequently, such an approach resulted in rather complex solutions.

6 Conclusions

In this paper, we explored the flow routing problem for two-tiered wireless sensor networks with the objective of maximizing network lifetime of upper-tier aggregation and forwarding nodes (AFNs). Existing flow routing solutions for maximizing network lifetime require AFNs to transmit flows to different nodes at the same time, which would require a packet-level power control to conserve energy. In this paper, we show that the packet-level power control is not necessary. Instead, it is possible to achieve the same maximum network lifetime by employing power control in a much larger timescale with the so-called single-session flow routing solutions. In addition, we show how to perform optimal single-session flow routing when the bit-rate generated by AFNs is time-varying, as long as the average bit-rate can be estimated. These results offer important understanding on lifetime-centric flow routing for energy-constrained wireless sensor networks.

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