Queueing Analysis of Sleep Management in an 802.15.4 Beacon Enabled PAN

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***Abstract*— The mean data rate obtained from a sensor field is one of the crucial quality of service parameters for sensor networks and related applications. In this work we consider the feasibility of duty cycle management as the vehicle to be used in order to achieve the desired data rate while maximizing the lifetime of an IEEE 802.15.4-compliant network in beacon enabled mode. We model and evaluate the duty cycle management algorithm using the theory of discrete time Markov chains and M/G/1/K queues with vacations. It is found that duty cycle management is capable of achieving almost 100% reliability and less than 1% utilization, under a wide range of packet arrival rates and network size.**

I. Introduction

Recently adopted IEEE 802.15.4 standard is poised to become the key enabler for low complexity, ultra low power consumption, low data rate wireless connectivity among inexpensive devices such as sensors [1]. Two network topologies are allowed by the standard, but both of them rely on the presence of a central controller device known as the PAN coordinator. In the peer-to-peer topology, devices can communicate with one another directly, as long as they are within the physical range. In the star-shaped topology, the devices must communicate through the PAN coordinator. The network uses two types of channel access mechanism, one based on slotted CSMA-CA algorithm in which the slots are aligned with the beacon frames sent periodically by the PAN coordinator, and another based on unslotted CSMA-CA similar to IEEE 802.11, in which case there are no beacon frames. The beacon enabled mode and the start-based hierarchical topology appear to be better suited to sensor network implementation then their unslotted or peer-to-peer counterparts because the PAN coordinator can act as both the network controller and the sink to collect the data from the sensor nodes.

Among the most important requirements for sensor networks is the maximization of their lifetime due to high costs (and sometimes even infeasibility) of maintenance activities. Sensor lifetime can be extended by adjusting the frequency and ratio of active and inactive periods of sensor nodes [2]. This approach is supported by the 802.15.4 standard in its beacon enabled mode with slotted CSMA-CA, where the interval between the two beacons is divided into active and inactive parts, and the sensors can switch to low-power mode during the inactive period. However, many sensor applications (such as surveillance, health care, and structural health monitoring) require continuous monitoring of relevant variables and events, and letting the network sleep for some time is simply out of the question. In such cases, duty cycle management can be applied at the level of individual sensor nodes, provided the number of sensors covering a given physical area is larger than the minimum number based on the required data rate. (As both cost and physical dimensions of wireless sensors are steadily decreasing, such a scheme is becoming increasingly attractive.) The desired data rate received at the sink (called event reliability in [2]) can then be achieved by adjusting the number of sensors that are active at any given time (this number is referred to as the sensor network QoS [3]). When redundant sensors are used, their duty cycle can be reduced to increase the lifetime of the network while maintaining the desired QoS. In this manner, the costly maintenance and human intervention can be reduced, which may well offset the increased initial cost of deployment. As an added bonus, the failure of individual sensors will not affect network performance since the duty cycle of the remaining ones can be increased to compensate for loss.

Only a few proposals to manage the number of active sensors in such networks have been reported to date. In [3], the authors propose a centralized approach in which the base station periodically collects packets from all sensors. It then determines whether the number of active sensors is sufficient to achieve the desired QoS and forwards this information over the broadcast channel to all sensors. The sensors independently determine, using the probabilistic Gur game automaton, whether to stay active or go to sleep. However, the number of

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active sensors is limited to one-half of the total number, due to the structure of the Gur automata. In addition, the energy efficiency is jeopardized by the fact that all sensors need to periodically report to the base station, regardless of their state, and they all need to listen to the transmissions from the base station even while in the low power state. (The energy consumption of the receiver is often nearly equivalent to that of the transmitter.)

In an alternative approach, each sensor transmits its data packets to the base station, which acknowledges it and provides a simple feedback on whether the actual QoS is higher than the desired one or not. The sensor then uses this information, through a simple probabilistic automaton, to independently determine whether to stay active or go to sleep [4]. Inactive nodes need not listen to transmissions from the base station and can go to sleep for prolonged periods. The energy consumption is thus reduced, and the overall network lifetime is increased compared to the approach from [3].

Unfortunately, neither of these approaches consider the real world environment in which the packets from different sensors can experience significant delays due to collisions and collision avoidance. As a consequence, the dependency between the number of sensors and arrival rates of sensor data, as independent variables, and the resulting reliability and lifetime, as dependent ones, will be much more complicated, and detailed analyses are needed to assess the feasibility of the duty cycle management approach.

A recent paper has investigated the performance of the IEEE 802.15.4 star topology network in the beacon-enabled mode [5]. The simulation-based analysis has revealed some of the key throughput-energy-delay tradeoffs inherent to the 802.15.4 MAC protocol, and highlighted the importance of the optimum choice of duty cycles and data rates.

In this paper, we analyze the performance of a redundant, duty-cycle-limited sensor network operating under IEEE 802.15.4 standard [1], using the theory of queues with vacations. We assume that the network operates in a beacon-enabled slotted CSMA-CA mode, and that each sensor has a small buffer for data packets obtained by sensing the target physical variable. When the sensor becomes active after a sleep period, it will try to send the data from its buffer to the base station – i.e., the PAN coordinator. After emptying the buffer, the sensor node goes to sleep for a random time which is geometrically distributed. If the buffer is empty when the sensor wakes up, it will immediately go to sleep again. We investigate the effect of the duration of the sleep period on the performance of the network; to that end, we model the network adapter of the sensor device as a M/G/1/K system with vacations. The server is, in fact, the MAC protocol which executes backoff procedures, does clear channel assessments, waits for and receives acknowledgements, and handles packet retransmissions in the case of collisions. The MAC protocol is modeled as a discrete time Markov chain with state transitions at the edges of backoff period boundaries. The model is, then, used to derive the expression for the mean number of packets received by the network coordinator; this variable is called "network reliability" in [2]. We show that the network reliability is a function of different MAC parameters, packet size, buffer size, network size (i.e., number of nodes) and of duty cycle management discipline. Furthermore, we derive expressions for the sensor utilization, mean number of active nodes, duration of sleep/vacation cycle, and duration of busy period per node. On the side of the MAC, we derive the channel access probability, the probability that the medium is idle on both clear channel assessments, and the probability distribution of the packet queue size.

The paper is organized as follows. In Section II, we present the queueing model for the sensor node with duty cycle management. In Section III, we model the behaviour of the MAC algorithm, using a discrete time Markov chain and the queueing model derived earlier. The probability distribution for the packet service time is calculated in Section IV. Section V presents and discusses the results of our analysis, and Section VI concludes the paper.

II. Determining the sensor utilization

In the discussion that follows we consider the management of sensor activity in the 802.15.4 network with beacon enabled, slotted CSMA-CA channel access mechanism. The probability distribution of the packet service time in the MAC sublayer may be described by the Probability Generating Function (PGF) $T_t(z)$. The Laplace-Stieltjes Transform (LST) of this probability distribution $T_t^*(s)$ can be obtained by substituting the variable z with e^{-s} in the expression for $T_t(z)$. Also, let the probability distribution function of the packet service time be denoted with $T_{dt}(x) = Prob[X \leq x]$, while the corresponding probability density function is denoted with $t_{dt}(x)$. Mean duration of the packet service time is, then, $\overline{T_t} = T_t'(1)$. We will also use the probability to access the medium, τ, as well as the probabilities that the medium is idle on the first and second Clear Channel Assessment (CCA). For the time being, we will assume that all the parameters of MAC

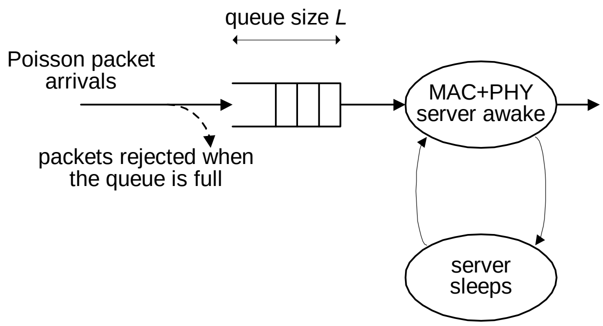


Fig. 1. M/G/1/K Queuing/vacation model for sensor node

sublayer are known; their values will be calculated in Sections III and IV.

Let n sensors be associated with the network coordinator. The buffer at the network interface of the device has the capacity to store L packets. The packet arrival process to sensor i is modeled as the Poisson process with arrival rate λ_i. The MAC sublayer will serve all packets in the queue in FCFS and exhaustive fashion, i.e., it will be active as long as there are packets in the device buffer according to the queueing model given in Fig. 1. When the buffer is emptied, the server (MAC) takes a vacation and the sensor goes to sleep. During the vacation period, packets continue to arrive to the buffer, but due to the small size of the buffer some of them will be discarded. If the buffer is empty upon returning from vacation, the server immediately takes a new vacation in order to prevent idle waiting (and reduce overall energy consumption). The duration of the vacation period is geometrically distributed with the parameter P_{sleep}:

$$V(z) = \sum_{k=1}^{\infty}(1-P_{sleep})P_{sleep}^{k-1}z^k = \frac{(1-P_{sleep})z}{1-zP_{sleep}} \quad (1)$$

We will now analyze the system, starting from Markov points which include moments of packet departure and moments when sensor wakes up from the vacation. We will make use of the Laplace-Stieltjes transform of the vacation time, $V^*(s)$, and the corresponding probability distribution function, $V(x)$, and probability density function, $v(x)$. Mean duration of the vacation is $\overline{V} = V'(1) = 1/P_{sleep}$.

The probabilities of k packet arrivals to the sensor buffer during the packet service time and during the sleep time are denoted as a_k and f_k, respectively, and they amount to

$$\begin{aligned} a_k &= \int_0^{\infty} \frac{(\lambda_i x)^k}{k!} e^{-\lambda_i x} t_{dt}(x)dx \\ f_k &= \int_0^{\infty} \frac{(\lambda_i x)^k}{k!} e^{-\lambda_i x} v(x)dx \end{aligned} \quad (2)$$

We also note [6] that the PGF for the number of packet arrivals to the sensor buffer during the packet service time and sleep time, respectively, are

$$\begin{aligned} A(z) &= \sum_{k=0}^{\infty} a_k z^k \\ &= T_t^*(\lambda_i - z\lambda_i) \\ F(z) &= \sum_{k=0}^{\infty} f_k z^k \\ &= V^*(\lambda_i - z\lambda_i) \end{aligned} \quad (3)$$

When the LSTs of the packet service time and vacation time are known, the probabilities a_k and f_k can be obtained as

$$\begin{aligned} a_k &= \left.\frac{1}{k!}\frac{d^k A(z)}{dz^k}\right|_{z=0} \\ f_k &= \left.\frac{1}{k!}\frac{d^k F(z)}{dz^k}\right|_{z=0} \end{aligned} \quad (4)$$

Let π_k and q_k denote the steady state probabilities that there are k packets in the device buffer immediately upon the departure of a packet and after returning from vacation, respectively. Then, the steady state equations for state transitions are

$$\begin{aligned} \pi_k &= \sum_{j=1}^{k+1}(q_j+\pi_j)a_{k-j+1}, \\ &\qquad \text{for } \ 0 \le k \le L-2 \\ \pi_{L-1} &= q_L + \sum_{j=1}^{L-1}(q_j+\pi_j)\sum_{k=L-j}^{\infty} a_k \\ q_k &= (q_0+\pi_0)f_k, \\ &\qquad \text{for } \ 0 \le k \le L-1 \\ q_L &= (q_0+\pi_0)\sum_{k=L}^{\infty} f_k \\ 1 &= \sum_{k=0}^{L} q_k + \sum_{k=0}^{L-1} \pi_k \end{aligned} \quad (5)$$

The probability distribution of the device queue length at the time of packet departure can be found by solving the system in a recursive manner. If we introduce the substitution $\pi'_k = \frac{q_k+\pi_k}{q_0+\pi_0}$, we obtain

$$\begin{aligned}\pi'_0 &= 1\\ \pi'_{k+1} &= \frac{1}{a_0}\left(\pi'_k - \sum_{j=1}^{k}\pi'_j a_{k-j+1} - f_k\right), \quad (6)\\ &\qquad\qquad \text{for } \ 0 \le k \le L-2\end{aligned}$$

From this system we can obtain the probability that the buffer is empty either upon a packet transmission or upon returning from sleep, i.e., the probability that the vacation follows the arbitrary Markov point, as

$$q_0 + \pi_0 = \frac{1}{\sum_{k=0}^{L-1}\pi'_k + \sum_{k=L}^{\infty} f_k} \quad (7)$$

Consequently, the probability that the packet service follows the arbitrary Markov point is $1 - q_0 - \pi_0$, and the mean distance between two Markov points is

$$\eta = (q_0 + \pi_0)\overline{V} + (1 - q_0 - \pi_0)\overline{T_t} \quad (8)$$

Then the probability that the sensor is active is

$$\rho' = \frac{(1 - q_0 - \pi_0)\overline{T_t}}{\eta} \quad (9)$$

We will utilize this parameter in the analysis of the operation of the MAC sublayer in Section III.

Offered load to the system is $\rho = \lambda_i\overline{T_t}$, while the probability that the packet will not be admitted to the sensor buffer due to insufficient space, is

$$P_B = 1 - \frac{\rho'}{\rho}, \quad (10)$$

Given that total number of sensors is n, the mean number of active sensors is

$$n_{on} = \sum_{k=0}^{n} k\binom{n}{k}\rho'^k(1-\rho')^{n-k} \quad (11)$$

The sensor network reliability per node – i.e., the number of packets that each sensor delivers to the coordinator per time unit – is equal to one over mean time between packet departures from the sensor. This time can be found by noting that the time between the start of two consecutive vacations (i.e., the vacation cycle) is

$$\overline{C_v} = \frac{\eta}{q_0 + \pi_0} \quad (12)$$

Of course, the vacation cycle is equal to the sum of the durations of busy period and subsequent vacation. Therefore, the duration of busy period is

$$\overline{\Theta_v} = \frac{\eta}{q_0 + \pi_0} - \overline{V} \quad (13)$$

The number of packets sent during the busy period is

$$n_a = \overline{\Theta_v}/\overline{T_t} \quad (14)$$

Then, the mean period between two packet transmissions is $C_v/n_a = \overline{T_t}t_{boff}/\rho'$, and the number of packets received by the coordinator is

$$r = \frac{\rho'}{\overline{T_t}t_{boff}} = \lambda_i(1 - P_B) \quad (15)$$

where t_{boff} denotes the duration of basic backoff period from MAC algorithm. Note that r corresponds to the network reliability as defined in [2], although it is, in fact, the throughput of the coordinator node in packets.

Assuming that the total length of physical layer headers and MAC headers and trailer is M bytes, and that the mean packet length at the MAC sublayer is G backoff periods, the throughput can be calculated as

$$S = nr(G - M/10) \quad (16)$$

This analysis brings up the challenging question: how to tune the sleep (vacation) time of the node in order to achieve required reliability at the coordinator. Mathematically speaking we need to solve the equation

$$nr = n\frac{1 - q_0 - \pi_0}{(q_0 + \pi_0)\overline{V} + (1 - q_0 - \pi_0)\overline{T_t}} = K \quad (17)$$

for P_{sleep}, given the desired network reliability K. As mentioned above, the packet service time in the MAC sublayer depends on the packet arrival rate, packet length, and the number of nodes. As individual nodes do not know the total number of nodes, they cannot know the total number of packets received by the network coordinator. In Section V, we will model and evaluate few policies which can control reliability per node.

III. Analysis of the MAC sublayer

Let us now determine the values of different parameters of the MAC layer. Note that the time needed to transmit a packet from the head of the queue includes the time from the moment when the CSMA-CA algorithm has started (i.e., from the start of the backoff countdown procedure) to the moment when the receipt of the packet has been acknowledged by the destination device. We assume that the MAC sublayer will retry the transmission of the packet until a positive acknowledgment is received.

The MAC can be modeled with discrete time Markov chain with the inclusion of vacation (sleep) states and transition probabilities from the energy management system, as shown in Fig. 2. The system enters the vacation state with the probability π_0 – which is the

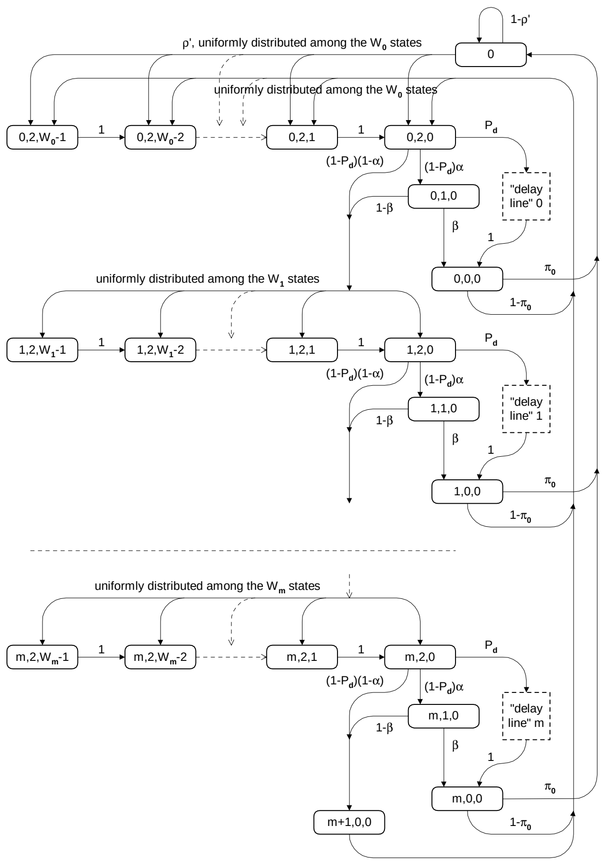


Fig. 2. Markov chain model of slotted CSMA-CA algorithm in non-saturation regime with finite buffer.

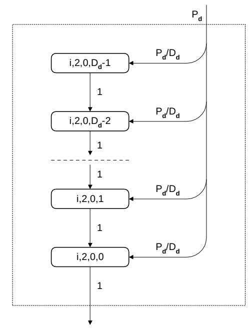


Fig. 3. Delay line for Fig. 2

probability that the device buffer is empty upon packet departure. The probability to stay in the vacation state (which means that the sensor remain inactive) is $1-\rho'$ (the value of ρ' has been calculated earlier).

Let us now introduce the following random variables: Let $b(t)$ represent the value of the backoff time counter which, at the beginning of the backoff period, can take any value in the range $0\,..\,2^{BE}-1$. When the counting starts, it decrements at the boundary of each backoff unit period. The value $b(t)$ will be frozen during the inactive portion of the beacon interval, and countdown will resume after the end of the beacon in the next superframe.

Let $n(t)$ represent the value of *NB* at time t; it belongs to the range $0\,..\,$*macMaxCSMABackoffs* $-\,1$.

Let $c(t)$ represent the value of *CW* at time t; it may be 0, 1, or 2.

Finally, let $d(t)$ represent the current value of the "delay line counter", which is started if the transmission can't be finished within the current superframe. Assuming a fixed packet size equal to G_p backoff periods (including MAC and PHY headers), the period between the packet and its acknowledgement equal to t_{ack} backoff periods, and the duration of the acknowledgement packet G_{ack}, the number of backoff periods necessary to complete the transmission within the current superframe is $D_d = 2 + G_p + t_{ack} + G_{ack}$ (we neglect the beacon frame size here). The probability that the remaining time within the superframe will not suffice to complete the transmission, i.e., that the MAC will have to go through the delay line, is $P_d = D_d/SD$, where SD denotes the superframe size.

The process $\{n(t), c(t), b(t), d(t)\}$ which defines the state of the device at backoff unit boundaries, is shown schematically in Figs. 2 and 3. To reduce the notational complexity, we have shown the last tuple member $d(t)$ only within the "delay line" and omit it in other cases where its value is zero. Also, the idle state with no packets to transmit is denoted only as state x_0. Also, the probabilities $P\{n(t+1) = i, c(t+1) = j, b(t+1) = k-1, d(t+1) = l-1 \mid n(t) = i, c(t) = j, b(t) = k, d(t) = l\}$ are written simply as $P\{i, j, k-1, l-1 \mid 0, j, k, l\}$. Finally, the constant *macMaxCSMABackoffs* which represents the maximum value of the variable *NB* is denoted with m.

For convenience, let W_0 stand for $2^{macMinBE}$, and let i represents the current value of *NB* during the execution of the algorithm ($i = 0\,..\,m$). Then, the maximum value of the random waiting time (expressed in units of backoff periods) that corresponds to i will be $W_i = W_0 2^{min(i, 5-macMinBE)}$.

The probabilities that the medium is idle on first and second CCA are denoted as α and β, respectively; they have to be distinguished because their values differ. Namely, at first CCA, the medium is busy if there is a transmission in progress (originating from another device), and this may be the start, middle, or end of that particular packet. At second CCA, however, the medium is busy only if some other device has just started its transmission (i.e., this must be the *first* backoff period of that packet).

The non-null transition probabilities can be described with the following:

$$
\begin{aligned}
P\{0,2,k \mid i,0,0\} &= \frac{1-\pi_0}{W_0}, \\
&\quad \text{for } i = 0 \,..\, m; k = 0 \,..\, 2^{BE}-1 \\
P\{0,2,k \mid 0\} &= \frac{\rho'}{W_0}, \\
&\quad \text{for } i = 0 \,..\, m; k = 0 \,..\, 2^{BE}-1 \\
P\{0 \mid i,0,0\} &= \pi_0, \\
&\quad \text{for } i = 0 \,..\, m \\
P\{0,2,k \mid m+1,0,0\} &= \frac{1}{W_0}, \\
&\quad \text{for } k = 0 \,..\, 2^{BE}-1 \\
P\{i,2,k-1 \mid i,2,k\} &= 1, \\
&\quad \text{for } i = 1 \,..\, m; k = 1 \,..\, 2^{BE}-1 \\
P\{i,1,0 \mid i,2,0\} &= \alpha(1-P_d), \\
&\quad \text{for } i = 0 \,..\, m \\
P\{i,0,0 \mid i,1,0\} &= \beta, \\
&\quad \text{for } i = 0 \,..\, m \\
P\{i+1,2,k \mid i,2,0\} &= \frac{(1-\alpha)(1-P_d)}{W_{i+1}}, \\
&\quad \text{for } i = 0 \,..\, m; k = 0 \,..\, 2^{BE}-1 \\
P\{i+1,2,k \mid i,1,0\} &= \frac{1-\beta}{W_{i+1}}, \\
&\quad \text{for } i = 0 \,..\, m; k = 0 \,..\, 2^{BE}-1 \\
P\{i,2,0,l \mid i,2,0\} &= \frac{P_d}{D_d}, \\
&\quad \text{for } i = 0 \,..\, m; l = 0 \,..\, D_d-1 \\
P\{i,2,0,l-1 \mid i,2,0,l\} &= 1, \\
&\quad \text{for } i = 0 \,..\, m; l = 0 \,..\, D_d-1 \\
P\{i,0,0 \mid i,2,0,0\} &= 1, \\
&\quad \text{for } i = 0 \,..\, m
\end{aligned}
\tag{18}
$$

Let the stationary distribution of the chain be $x_{i,j,k,l} = \lim_{t\to\infty} P\{n(t) = i, c(t) = j, b(t) = k, d(t) = l\}$, for $i = 0 \,..\, m; j = 0,1,2; k = 0 \,..\, 2^{BE}-1; l = 0 \,..\, D_d - 1$. For brevity, we will omit l whenever it is zero, and introduce the auxiliary variables C_1, C_2, and C_3:

$$
\begin{aligned}
x_{0,1,0} &= x_{0,2,0}(1-P_d)\alpha \\
&= x_{0,2,0}C_1 && (19) \\
x_{1,2,0} &= x_{0,2,0}(1-P_d)(1-\alpha\beta) \\
&= x_{0,2,0}C_2 && (20) \\
x_{0,0,0} &= x_{0,2,0}((1-P_d)\alpha\beta + P_d) \\
&= x_{0,2,0}C_3 && (21)
\end{aligned}
$$

Then, we obtain the following relations:

$$
\begin{aligned}
x_0 &= x_{0,0,0}\frac{\pi_0\left(1-C_2^{m+1}\right)}{\rho'(1-C_2)} \\
x_{i,0,0} &= x_{0,0,0}C_2^i, \\
&\quad \text{for } i = 0 \,..\, m \\
x_{i,2,k} &= x_{0,0,0}\frac{W_i - k}{W_i}\cdot\frac{C_2^i}{C_3}, \\
&\quad \text{for } i = 1 \,..\, m; k = 0 \,..\, W_i - 1 \\
x_{i,1,0} &= x_{0,0,0}\frac{C_1C_2^i}{C_3}, \\
&\quad \text{for } i = 0 \,..\, m \\
x_{0,2,k} &= x_{0,0,0}\frac{W_0 - k}{W_0\ C_3} \\
&\quad \text{for } k = 1 \,..\, W_0 - 1 \\
x_{m+1,0,0} &= x_{0,0,0}\frac{C_2^{m+1}}{C_3} \\
\sum_{l=0}^{D_d-1} x_{i,2,0,l} &= \frac{x_{0,0,0}C_2^iP_d(D_d-1)}{2C_3}
\end{aligned}
\tag{22}
$$

Since the sum of all probabilities in the Markov chain must be equal to one, we obtain:

$$
\begin{aligned}
x_0 &+ \sum_{i=0}^{m}\sum_{k=0}^{W_i-1} x_{i,2,k} + \sum_{i=0}^{m} x_{i,0,0} + \sum_{i=0}^{m} x_{i,1,0} \\
&+ x_{m+1,0,0} + \sum_{i=0}^{m}\sum_{l=0}^{D_d-1} x_{i,2,0,l} = 1
\end{aligned}
\tag{23}
$$

which gives

$$
\begin{aligned}
\frac{1}{x_{0,0,0}} &= \sum_{i=0}^{m}\frac{C_2^i(W_i+1)}{2C_3} + \frac{C_2^{m+1}}{C_3} \\
&+ \frac{1-C_2^{m+1}}{1-C_2}\left(1 + \frac{C_1}{C_3} + \frac{\pi_0}{\rho'} + \frac{P_d(D_d-1)}{2C_3}\right)
\end{aligned}
\tag{24}
$$

The total probability to access the medium is

$$
\tau = \sum_{i=0}^{m} x_{i,0,0} = x_{0,0,0}\frac{1-C_2^{m+1}}{1-C_2} \tag{25}
$$

However, we should distinguish between the probability τ_1 to access the medium when the transmission is deferred to the next superframe due to lack of space, and the probability τ_2 to access the medium in the current superframe:

$$
\begin{aligned}
\tau_1 &= \frac{P_d}{C_3}\tau \\
\tau_2 &= \left(1 - \frac{P_d}{C_3}\right)\tau
\end{aligned}
\tag{26}
$$

We now estimate probabilities that medium is idle on the first and second CCA. At any moment, k_{on} stations out of n can be active ($0 \leq k_{on} \leq n$). The number of active stations follows the binomial distribution, i.e. $P(k_{on}) = \binom{n}{k_{on}} \rho'^{k_{on}} (1-\rho')^{n-k_{on}}$.

Exactly q active stations (where $q = 0 \,..\, k_{on}$) will have transmissions delayed to the beginning of the next superframe. We assume that the variables q and $k_{on} - 1 - q$, which denote non-delayed and delayed packet transmissions, respectively, follow a binomial distribution with probability $P_q(k_{on}) = \binom{k_{on}-1}{q} (1 - P_d)^q P_d^{k_{on}-1-q}$.

The probability α that the medium is idle at the first CCA is obtained as the ratio of the mean number of busy backoff periods within the superframe and the total number of backoff periods in the superframe, SD. The probability that one or more delayed packets will be transmitted in one superframe will take place is

$$n_1(k_{on}) = (1 - (1-\tau_1)^{(k_{on}-1-q)}) \tag{27}$$

and the number of busy backoff periods due to these transmissions is $n_1(k_{on})(G_p + G_{ack})$.

As the mean period between two transmissions of non-delayed packets is $1/(q\tau_2)$, the mean number of non-delayed packet transmissions within the superframe is

$$\begin{aligned} n_2(k_{on}) &= (1 - (1-\tau_1)^{(k_{on}-1-q)})(SD - 2Dd)q\tau_2 \\ &+ (1-\tau_1)^{(k_{on}-1-q)}(SD - D_d)q\tau_2 \end{aligned} \tag{28}$$

Finally, we note that first two CCA evaluations after the beacon will be positive and we will denote this factor as $\alpha_0 = 2/(SD - D_d)$. Then, the probability that the medium is idle at the first CCA is

$$\alpha = 1 - (1-\alpha_0) \sum_{k_{on}=1}^{n} \binom{n}{k_{on}} \rho'^{k_{on}} (1-\rho')^{n-k_{on}} PQ \tag{29}$$

where

$$PQ = \sum_{q=0}^{k_{on}-1} P_q(k_{on}) \frac{(n_1(k_{on}) + n_2(k_{on}))(G_p + t_{ack} + G_{ack})}{SD - D_d} \tag{30}$$

Then, the probability that medium is idle at the second CCA is

$$\begin{aligned} \beta = &\binom{n-1}{0}(1-\rho')^{n-1} + \\ &\sum_{k_{on}=1}^{n} \binom{n}{k_{on}} \rho'^{k_{on}} (1-\rho')^{n-k_{on}} \times \\ &\times \sum_{q=0}^{k_{on}-1} P_q(k_{on}) \left(\frac{1 + (1-\tau)^{(k_{on}-1)}}{SD - D_d} + \right. \\ &\frac{(Dd-2)}{SD - D_d}(1-\tau_1)^{(k_{on}-1-q)}(1-\tau_2)^q \\ &\left. + \frac{SD - 2D_d}{SD - D_d}(1-\tau_2)^q \right) + (1-\alpha+\alpha_0)\frac{t_{ack}-1}{D_d} \end{aligned} \tag{31}$$

where the last component comes from the possible positive second CCA during the gap between the packet and its acknowledgement; note that $1 - \alpha + \alpha_0$ denotes the probability of a transmission going on.

IV. Determining the Probability Distribution for the Packet Service Time

We will assume that every transmitted packet has to be acknowledged, otherwise packets have to be retransmitted. Let us denote the PGF of the packet length as $G_p(z)$ and the mean packet size as $G = G_p'(1)$ backoff periods. Let the PGF of the interval between the end of the packet and the arrival of the corresponding acknowledgement (*macMaxAckWait*) be $S_{ack}(z)$, and let $G_{pa}(z)$ stand for the PGF of the acknowledgement duration. The PGF of the duration of the beacon frame is denoted as $B_{ea}(z)$. We will also use the MAC parameter *BO* (which stands for *macBeaconOrder*) that regulates the period between the beacons as

$$BI = 2^{BO} \mathit{aBaseSuperframeDuration} \tag{32}$$

The duration of the superframe depends on the energy management policy of the network. For simplicity, we assume that *BO* has a constant value of zero, in which case the superframe duration is

$$SD = \mathit{aBaseSuperframeDuration} \tag{33}$$

We will need the probability distribution of the packet service time at the MAC sublayer, which is equal to the service time for the packet queue at the network node. In order to derive this distribution, we begin by modeling the effect of freezing the backoff counter during the inactive period of the superframe. The probability that a backoff period is the last one within the active superframe is

$$P_{last} = \frac{1}{SD} \tag{34}$$

The PGF for the effective duration of the backoff period, including the duration of the beacon frame, is

$$B_{off}(z) = (1-P_{last})z + P_{last} z^{(BI-SD+1)} B_{ea}(z) \quad (35)$$

The PGF for the duration of i-th backoff attempt is

$$B_i(z) = \sum_{k=0}^{W_i-1} \frac{1}{W_i} B_{off}^k(z) = \frac{B_{off}^{W_i}(z) - 1}{W_i(B_{off}(z) - 1)} \quad (36)$$

The transmission procedure will not start unless it can be finished within the current superframe, as explained above. The probability that the remaining number of backoff periods will not suffice for the transmission has been defined in Section III as

$$P_d = \frac{D_d}{SD} \quad (37)$$

where $D_d = 2 + G_p'(1) + S_{ack}'(1) + G_{pa}'(1)$ is the number of backoff periods needed for an acknowledged transmission. The PGF for the number of backoff periods that are wasted due to the insufficient space in the current superframe is

$$B_p(z) = \frac{1}{D_d} \sum_{k=0}^{D_d-1} z^k \quad (38)$$

The PGF of the packet transmission time is

$$\begin{aligned} T_{ran}(z) &= (1-P_d)G_p(z)S_{ack}(z)G_{pa}(z) \\ &+ P_d B_p(z) z^{(BI-SD)} B_{ea}(z) G_p(z) S_{ack}(z) G_{pa}(z) \end{aligned} \quad (39)$$

Then, the PGF of the packet service time $T_t(z)$, accounting for the effects of backoffs, unsuccessful CCAs, transmission, carry-over to the next superframe, and re-transmission, satisfies the equation

$$\begin{aligned} T_t(z) &= \sum_{i=0}^{m} \prod_{j=1}^{i+1} B_{j-1}(z) \left(1-\alpha\beta\right)^i \alpha\beta^2 T_{ran}(z) \\ &+ \sum_{i=0}^{m} \prod_{j=1}^{i+1} B_{j-1}(z) \left(1-\alpha\beta\right)^i \alpha\beta(1-\beta) T_{ran}(z) T_t(z) \\ &+ \left(1 - \alpha\beta \sum_{i=0}^{m} \left(1-\alpha\beta\right)^i \right) \prod_{j=0}^{m} B_j(z) T_t(z) \end{aligned} \quad (40)$$

This equation has been derived under the assumption that the MAC sublayer will retry packet transmission until the acknowledgement is received. As the standard currently allows a maximum of three retries, our calculations will tend to overestimate the packet service time under high loads. As the 802.15.4 PAN is much more likely to operate under low to moderate loads, the error due to this difference will be negligible in practice.

The last equation can be rearranged to give the PGF for the packet service time as

$$T_t(z) = \frac{\sum_{i=0}^{m} \prod_{j=1}^{i+1} B_{j-1}(z)(1-\alpha\beta)^i \alpha\beta^2 T_{ran}(z)}{DT} \quad (41)$$

where

$$\begin{aligned} DT &= 1 - \sum_{i=0}^{m} \prod_{j=1}^{i+1} B_{j-1}(z)(1-\alpha\beta)^i \alpha\beta(1-\beta) T_{ran}(z) \\ &- \left(1 - \alpha\beta \sum_{i=0}^{m} (1-\alpha\beta)^i \right) \prod_{j=0}^{m} B_j(z) \end{aligned} \quad (42)$$

The first two moments of the packet service time can be obtained as $\overline{T_t} = T_t'(1)$ and $\overline{T_t^{(2)}} = T_t''(1) + T_t'(1)$.

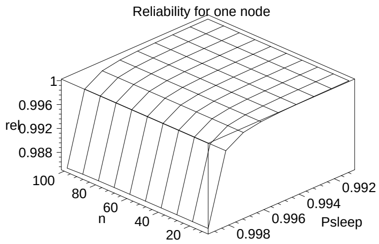
V. Analytical results for traffic performance in PAN

To demonstrate the feasibility of our analytical approach, we have evaluated the network reliability and QoS (using the definitions in [2] and [3], respectively) under varying values of different network and traffic parameters. The packet size has been fixed at $G = 9$ backoff periods, while the device buffer had a fixed size of $L = 3$ packets. We have assumed exhaustive service discipline, i.e., the device will deliver all the packets from its buffer before going to sleep. Moreover, the new packets that arrive during the active period, will also be serviced (i.e., sent to the coordinator) before going to sleep.

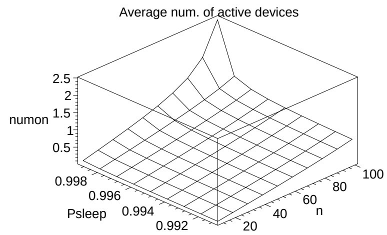
In all calculations we have considered networks that operate in the ISM band at 2.4GHz, with raw data rate 250kbps and *SO*,*BO*$= 0$. In that case, one modulation symbol corresponds to four data bits, *aUnitBackoffPeriod* has 10 bytes, while *aBaseSlotDuration* has 30 bytes; as *aNumSuperframeSlots* is 16, the *aBaseSuperframeDuration* is exactly 480 bytes. Furthermore, we have assumed the following values for the parameters of the CSMA-CA MAC algorithm:

1) The minimum value of backoff exponent *macMinBE* is set to three.
2) The maximum value of backoff exponent *aMaxBE* is set to five.
3) The maximum number of backoff attempts is set to five, *macMaxCSMABackoffs* $= 4$.

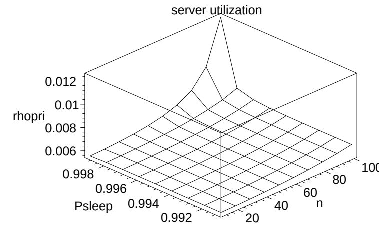
The packet size includes all physical layer and MAC sublayer headers, and is expressed as the multiple of the backoff period [1]. We also assume that the physical layer header has 6 bytes, and that the MAC sublayer header and Frame Check Sequence fields have



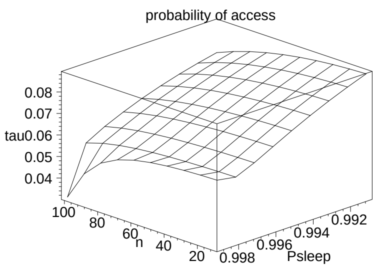
(a) Mean number of transmitted packets (reliability) per node.



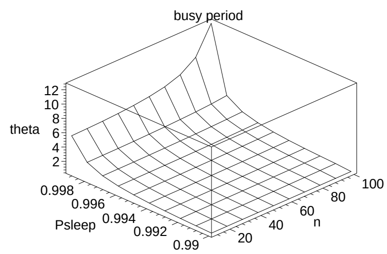
(b) Mean number of active nodes (QoS).



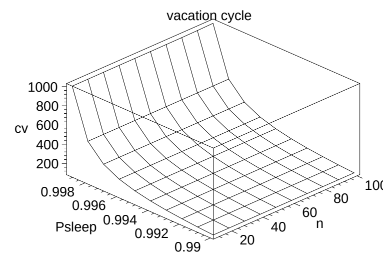
(c) Server utilization.



(d) Access probability at each node.



(e) Mean duration of the busy period of a node.



(f) Mean duration of the vacation cycle for each node.

Fig. 4. Network performance under variable P_{sleep} (periods are shown as multiples of backoff periods).

a total of 9 bytes. Such a short MAC header implies that the destination addressing mode subfield (bits 10-11) within frame control field is set to 0, and source addressing mode field (bits 14-15) is set to short address mode. This means that packet is directed to the PAN coordinator with the PAN identifier as specified in the source PAN identifier field.

Our first set of experiments considers the network in which the arrival rate at each sensor has been fixed at one packet per second, while the probability of going to sleep P_{sleep} was variable. The number of sensor nodes has been varied in the range of 10 to 100. The resulting performance data is shown in Fig. 4.

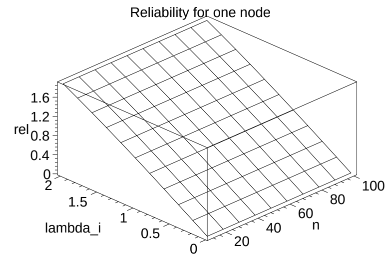
As can be seen, the reliability per node is close to one, except at very high values of the probability P_{sleep}, in which case the vacation periods last too long, as shown in Fig. 4(f), the device buffer fills up, and packets are not even accepted at the device. At the same time, server utilization remains below 1% in a wide range of values for n and P_{sleep}, which means that the network is fairly energy-efficient. (Note that a single AAA battery can power an off-the-shelf radio transceiver at $10mA$ for two years if duty cycle below 0.5% is maintained [7].)

From Fig. 4(e) we observe that the busy period increases with the number of nodes because of higher probability of collisions. It also increases with P_{sleep} because longer sleep increases the probability that the device will have a full buffer to deliver. Steep increase in the far end of the diagram is due to the increase in number of packets to deliver – namely, the packets that are in the buffer at wake-up take longer to be serviced, which means there is more time for new ones to arrive while the server is active, in which case they have to be serviced.

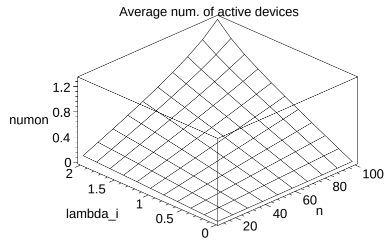
Then, we have fixed the variable P_{sleep} to the value of 0.999, and let the packet arrival rate and the number of nodes vary; other parameters had the same values as before. The resulting diagrams are shown in Fig. 5. As can be seen, reliability per node and, by extension, the total network reliability, are almost linear functions of the packet arrival rate. Similar observation holds for server utilization and server busy period. It may be interesting to note that the access probability has a minimum for a certain packet arrival rate, and then slowly increases, while being nearly independent of the number of nodes. Again, we observe that the reliability is high while the server utilization is low (below 1% for packet arrival rates below two packets per second).

VI. Conclusion

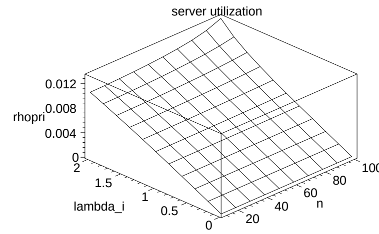
In this work we have modeled the reliability (or network QoS) of the sensor network based on the IEEE 802.15.4 standard, taking the features of the MAC sublayer into account. We have assumed that the duty cycle of each device is managed in such a way that



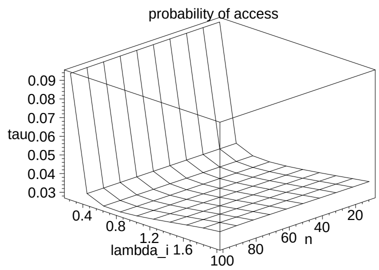
(a) Reliability for $P_{sleep} = 0.999$.



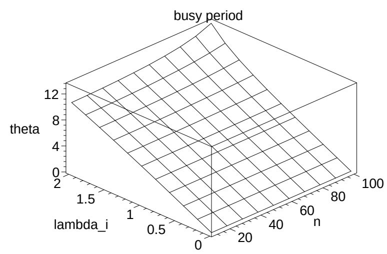
(b) Average number of active stations for $P_{sleep} = 0.999$.



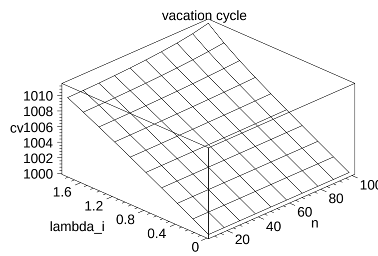
(c) Server utilization for $P_{sleep} = 0.999$.



(d) Access probability for $P_{sleep} = 0.999$.



(e) Average duration of node busy period for $P_{sleep} = 0.999$.



(f) Average duration of node vacation cycle for $P_{sleep} = 0.999$.

Fig. 5. Network performance with fixed $P_{sleep} = 0.999$ (periods are shown as multiples of backoff periods).

the device goes to sleep for a geometrically distributed random time when its buffer becomes empty. When the device is active, its buffer is serviced in an exhaustive manner, and packets that have arrived are serviced as well. We have shown that the network reliability is a function of the MAC backoff parameters, packet size, buffer size, network size and of the duty cycle management discipline. We have evaluated the network QoS under variable sleep parameters as well as under variable packet arrival rates per node and network size. It has been shown that node utilization ess than 1% and reliability higher than 99% under wide range of network size and packet arrival rate.

These results support the claim that the sensor duty cycle can be controlled to maintain the network reliability and QoS at the desired level while trying to minimize the energy consumption. In fact, an adaptive approach should exercise such control in real time. However, the feasibility of such an approach dictates that computations of probability distributions be avoided, due to limited computational capabilities of the PAN coordinator nodes. We are currently working on designing sleep disciplines, possibly with centralized control, that will further decrease the node utilization while maintaining reliability at the desired level.

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