End-to-End Burst Loss Probabilities in an OBS Network with Simultaneous Link Possession *

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Abstract

In this paper, we calculate analytically the end-to-end burst loss probabilities over a path in an Optical Burst Switched (OBS) network. To this effect, we consider OBS nodes linked by WDM fibers in series so as to form a single path. The traffic over this OBS path is generated by N end-devices. We consider large bursts, which are assumed to be long enough to occupy wavelengths on two successive OBS fibers (links) at the same time. A burst is dropped if all the wavelengths of a link are busy.

This OBS path is modeled by a queueing network with dynamic simultaneous resource possession. The burst arrival process from each transmitting OBS end-device is modeled by an IDLE-ON traffic source, which is represented by a two-state Markov process. In addition, Poisson-distributed local traffic is used in order to increase the congestion of each link in the path. The queueing network is analyzed approximately by a decomposition algorithm in order to calculate the burst loss probabilities at each link of the OBS path. The approximate results were verified by simulation for a variety of input parameters and the approximation algorithm was found to have a good accuracy.

1. Introduction

OBS is a viable solution for the next generation all-optical networks because it clearly separates its data and control planes. The user data and the corresponding control information are transmitted separately in time and space through the network. The control information is transmitted prior to the user data and it is electronically processed at each node along the route. The user data itself travels transparently as an optical signal from the source to the destination. This is advantageous because the OBS nodes just switch the optical signals without having to know any specific information about the format or the transmission rate of the user data.

In an OBS network, the data plane consists of the upper layer traffic collected by the OBS end-devices. This traffic is collected, sorted based on its destination address and it is assembled into variable size data units, called *bursts*. For each burst, the OBS end-devices construct a *control packet*, which is transmitted an *offset* time prior to the transmission of the burst. The control packet travels along the route and it reserves bandwidth resources for its corresponding burst at each OBS node. Upon receipt of the control packet, an OBS node assigns a free wavelength on the desired output port and configures its switching fabric to transparently switch the upcoming burst. After the *offset* time, the OBS end-device transmits the burst itself.

Because of the predicted high data rates, most OBS architectures support a *one-way reservation* scheme in order to decrease the end-to-end transmission delay. In other words, an end-device transmits a burst without a positive acknowledgment from the destination that the optical path has been set up. A burst may be dropped if it arrives at an OBS node, where the control packet was unsuccessful at reserving a wavelength on its desired output port. Therefore the calculation of the burst loss probability is an important measure of the performance of an OBS network.

Many performance evaluation studies of OBS have been reported in the literature. For example, the JET-based OBS protocol by Qiao and Yoo [9] has been studied by focusing on a single output port of an OBS node and assuming Poisson arrivals and full wavelength conversion (see Dolzer et al. [3],Yoo et al. [16],Vu et al. [12]). A single output port of an OBS node is modeled as an M/G/W loss system where W is the number of wavelengths per fiber. The burst loss probability is calculated using the well-known Erlang's B formula.

Other OBS studies consider the JET protocol in an OBS node with fiber delay lines (FDLs). The reason is that an FDL can buffer the optical signals for a small amount of time in case of contention at an output port and thus reduce the probability of burst loss. Yoo et al. [16] derived a lower bound for the blocking probability by approximating an OBS output port with FDLs as an M/M/W/D, where W is the number of wavelengths per fiber, $D = W + W * N$ and N is the number of FDLs. More recently, Lu and Mark [6] proposed a Markovian model for an OBS port with FDLs, which captures the bounded delay and the balking properties of FDLs.

Deflection routing has also been proposed as a strategy for contention resolution in a JET-based OBS network. Hsu et al. [5] proposed a two-stage Markovian model that approximates the behavior of a single output port of an OBS node with deflection routing. Chen et al. [1] also proposed Markovian models for deflection routing, but theirs are more general and could be applied to an OBS node with any number of output ports.

The optical composite burst switching (OCBS) [2] is another OBS variant. In OCBS, in case of contention only the initial part of the burst is dropped until a free wavelength becomes available. Therefore, in OCBS the loss probability is calculated in terms of the upper layer packets rather than the OBS bursts. Detti et al. [2] developed an analytical model for OCBS with an ON-OFF arrival process. Neuts et al. [8] also analyzed OCBS but they assumed a Poisson-distributed traffic arrivals which allowed them to use an M/G/∞ model.

The JIT OBS signaling protocol [13] was analyzed by Xu et al. [15] using a closed queueing network model for an edge OBS node where the arrival process was a 3-state Markovian model, which allowed for short and long bursts. Xu et al. [14] also analyzed the same queueing network assuming a large number of wavelengths.

Note that all of the previously cited analytical models focus on a single OBS node. These models provide a limited insight about the overall performance of an OBS network. To our knowledge, the only published analytical model of an OBS network is the one by Rosberg et al. [10]. Their approach is based on the reduced load fixed point approximation, where the load to each link is approximated by considering only the reduced load caused by the blocking at the previous links along the route. They consider a Poisson-distributed arrival process and assume that each burst occupies simultaneously a single-wavelength from each link along its route until it is lost or until it departs from the network.

In OBS, however, the size of the bursts can vary and also the distance between two adjacent nodes will vary depending on the network's topology. In view of this, it is unknown how many links each burst will occupy as it travels through the network. Short bursts may occupy a single link at a time but it is possible that large bursts can *simultaneously* occupy two or more links. This behavior differs from the widely studied packet-switched or circuit switched networks. In packet-switching, a packet typically occupies a small fraction of a link in a wide-area network, due to to the high-speed of the links and the relatively small size of a packet. On the contrary, in a circuit-switched network, such as the telephone network, a call simultaneously occupies one time slot on each link of the path between the source and the destination. Therefore, new techniques have be developed in order to investigate the performance of OBS networks, where a burst may occupy more than one link, but not all the links along the path between its source and its destination.

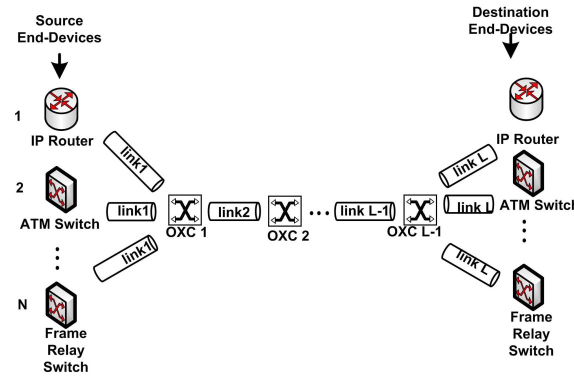

Figure 1. Tandem OBS Network

In this paper we study for the first time an OBS network where a burst occupies *simultaneously* more than one link, but not all links, along its route. We propose a new queueing model, which models dynamic simultaneous link possession. We also propose a burst arrival process, called IDLE-ON, which accurately captures the burst transmission at the edge of the OBS network. Most other analytical studies of OBS assume a Poisson arrival process. Our arrival process is more realistic because it reflects the fact that a new burst can not arrive into the network until the previous burst is completely transmitted.

The remainder of this paper is organized as follows. In Section 2, we describe the OBS network under study and in Section 3, we present a queueing network model of it and we describe the burst arrival process. In Section 4, we present a decomposition algorithm for analyzing this queueing network and in Section 5 we validate our algorithm against simulation and discuss the results. We conclude this paper in Section 6.

2. The OBS Network Under Study

We consider an OBS network, where the core nodes include optical crossconnects (OXC). An OXC can optically switch a burst on an incoming wavelength w_1 of an input port i to the same wavelength w_1 of any output port j. Each OXC output port has a full wavelength conversion capability. That is, in the above example, if wavelength w_1 of the output port j is busy then the burst will be converted into the frequency of any other free wavelength. The burst will be dropped if all wavelengths of the output port j are busy.

We analyze the performance of a specific path in this OBS network and thus we consider OXCs connected in tandem, as shown in Figure 1. Two adjacent OXCs are linked by a single WDM fiber. Each fiber has W transmission wavelengths. There are N OBS end-devices, which are connected to the first OXC via N separate links, each referred to as link 1. These end-devices collect upper layer traffic, sort it based on a destination address and assemble it into variable-size bursts. Similarly, there is a number of OBS *destination* end-devices, which are connected via separate links to OXC $L-1$, each referred to as link L.

We investigate large bursts which occupy more than one link at the same time as they travel through the network. Such bursts are possible since both the burst size and the link lengths are variable. As shown by the study of Madamopoulos et al. [7], in an U.S. metropolitan network of ~3600 km^2 the core rings have a a circumference of 50-250 km and are made of several OXCs connected by short links. In one of their scenarios, the circumference of a core ring is only 20.5 km and it is made of 4 OXCs, which results in an average link length of only ~4km. Assuming that the data rate is OC192, in this example all bursts larger than ~24KB will simultaneously hold resources on more than one link. In general, depending on the maximum burst size, the minimum link lengths and the data rate, bursts may span two, three or even more links at the same time.

In this paper, we focus on a network where bursts occupy wavelengths on two consecutive links simultaneously but we note that our algorithm can be modified to solve OBS networks where bursts span more than two links at a time. In the specific example from Figure 1, we assume that a burst launched by an end-device will first occupy a wavelength on link 1 and a wavelength on link 2 simultaneously, then it will move to occupy the same wavelength on link 2 and a wavelength on link 3 simultaneously, and so on until it departs from the network.

In an OBS network, there will be a mix of bursts that hold resources on single links and bursts that simultaneously hold resources on multiple links. The algorithm presented in this paper, though limited to the case where a bursts occupies exactly two bursts, provides a valuable first insight into the performance of an OBS network with simultaneous link possession. It is also a first step towards our goal to analyze a queueing model that will take into account both types of bursts.

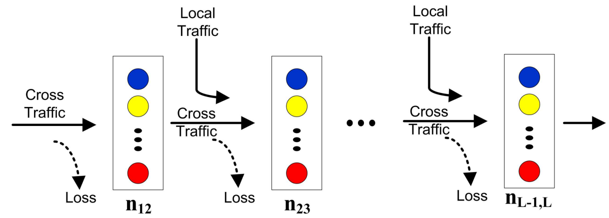

Figure 2. The Queueing Network Model

3. The Queueing Network Model

The queueing model that we propose for an OBS path with dynamic simultaneous possession of two links is shown in Figure 2. Since there is no buffering in OBS, the queueing network model does *not* contain any queues and therefore burst loss is possible at each link. The queueing network consists of a number of *loss* nodes linked in tandem. Each node consists of W servers, where W is the number of wavelengths on each link.

3.1. Modeling Simultaneous Possession

Each loss node of the queueing network from Figure 2 represents two adjacent links of the OBS path. That is, node 12 represents links 1 and 2, node 23 represents links 2 and 3 and so on. The number of bursts $n_{i-1,i}$ in loss node $(i-1, i)$ represents the number of bursts currently occupying links $(i-1)$ and link i. For example, if $n_{12} = 2$ and $n_{23} = 1$ then there are two bursts currently being transmitted over link 1 and link 2 and one burst on link 2 and link 3, see Figure 3. All three bursts are currently occupying a wavelength on link 2. Note that a burst frees up a wavelength on a link as soon as its tail departs an OXC. For instance, in Figure 3 despite the fact that the tail of burst 3 is still in link 1, a new burst can enter link 1 at the same wavelength. Similarly, as soon as the head of a burst enters a link then its assigned wavelength will be occupied for the duration of the burst. We note that a customer in our queueing network

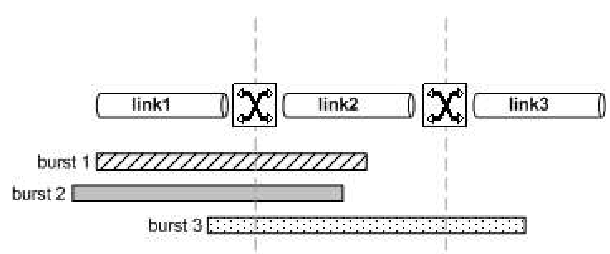

Figure 3. Burst occupation of links

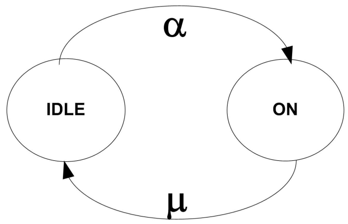

Figure 4. IDLE/ON Source

represents a burst, which always occupies a wavelength on two adjacent links at the same time. As the burst propagates through the network, the corresponding customer simply moves from one loss node of the queueing network to the next. Due to the full wavelength conversion capability at each OBS node, a burst may occupy one wavelength on link $(i-1)$ and the same or different wavelength on link i.

The maximum capacity of each queueing node is W, which means that no more than W burst can be transmitted over the same link at the same time. Therefore, we have the following constraint:

$$n_{i,i+1} \leq W \qquad for\ 1 \leq i \leq L-1 \tag{1}$$

Furthermore, each link of the network, except the first and the last one, is represented in two consecutive nodes of the queueing network. For example, node $(i-1, i)$ and node $(i, i+1)$ both contain link i, which can not transmit more than W bursts at one time. Therefore, the following constraint is also true:

$$n_{i-1,i} + n_{i,i+1} \leq W \qquad for\ 2 \leq i \leq L-1 \tag{2}$$

The burst length is exponentially distributed with a mean of $1/\mu$. It is possible to model the burst length with other distributions, such as Coxian or Phase-Type, but this will increase the dimensionality of the state space of the queueing network.

3.2. The Arrival Process

In the considered OBS network, each end-device is equipped with a single tunable transmitter, which can transmit on any of the W wavelengths of link 1. An end-device which is currently transmitting a burst, has to wait until the current transmission is completely finished before it can start the transmission of the next burst. Therefore, the interarrival time between bursts transmitted by the same end-device must be dependent on the time it takes to transmit a burst. If we model the arrival process as Poisson, bursts would arrive randomly and a new burst could arrive before the transmission of the previous burst has ended. This, however, is unrealistic because the end-devices have only a single transmitter. Instead, we model the burst arrival process with an IDLE-ON process, which is the two-state Markov process shown in Figure 4. An OBS end-device is in the ON state when it is transmitting a burst. It remains in this state for the duration of the burst, which is exponentially distributed with a mean of $1/\mu$. If the OBS end-device is not transmitting then it is in the IDLE state, which is also exponentially distributed with a mean of $1/\alpha$. Note, that the IDLE-ON process differs from the popular ON-OFF process (see Frost and Melamed [4]), used to model the arrival of packets. In IDLE-ON, the source transmits only *one* burst in the ON state and then it moves to the IDLE state. In the ON-OFF model, however, the source continuously transmits packets for the entire time it is in the ON state.

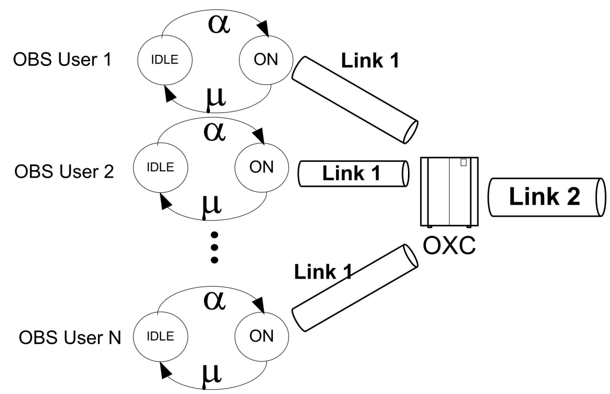

Figure 5. Multiplexed IDLE-ON Sources

The total traffic due to the N end-devices, from here on referred to as the *cross* traffic, is a multiplexed stream of the burst arrivals from all N OBS end-devices. That is, there are N IDLE-ON traffic sources, which generate traffic as shown in Figure 5. We assume that all N end-devices are modeled with an identical IDLE-ON process, i.e., each end-device has the same μ and α parameters. As mentioned previously, the bursts, generated by these sources, will occupy two links at once. Therefore, immediately upon entering the network, a burst will request a wavelength on link 2 and if no free wavelength is available then it will be dropped. Furthermore, only W OBS end-devices can transmit simultaneously because even though the end-devices own the entire bandwidth on their individual link 1, they all share the capacity of link 2. Recall that n_{12} indicates the number of bursts currently being transmitted over link 1 and 2, which means that n_{12} OBS end-devices are in the ON state and the remaining $(N - n_{12})$ end-devices are in the IDLE state. At that moment a new burst can only arrive from the $(N - n_{12})$

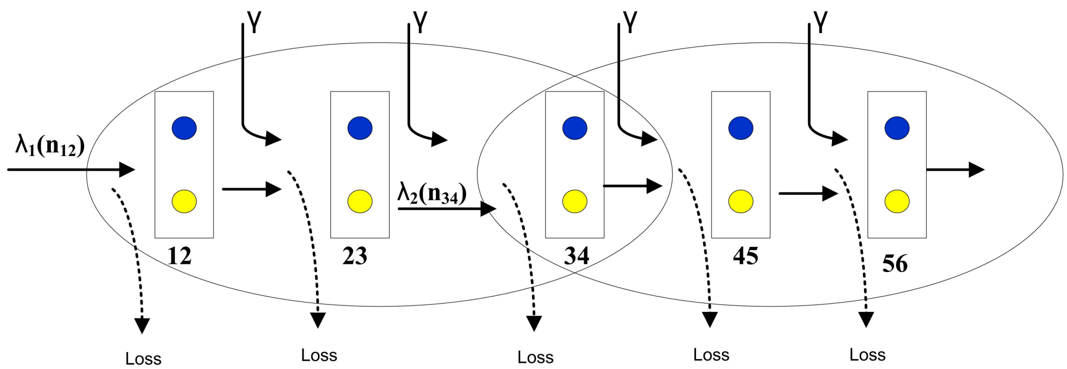

Figure 6. Queueing Network for an OBS path of 6 fibers with 2 wavelengths per fiber

sources that are currently not transmitting. That is because the n_{12} transmitting sources have to wait for the current transmission to be completed before attempting to transmit a new burst. In view of this, the arrival rate of the multiplexed arrival stream from the N OBS end-devices is:

$$\lambda_1(n_{12}) = (N - n_{12})\alpha, \qquad 0 \leq n_{12} \leq W \qquad (3)$$

In addition to the *cross* traffic, we also consider Poisson-distributed *local* traffic which loads the OBS path with extra bursts. This traffic arrives at each link, except the first one, and it subsequently becomes part of the *cross* traffic. That is, a *local* traffic burst enters the OBS path at an intermediate link and it is routed toward one of the destination end-devices.

Finally, we note that we can also consider the case where each end-device is equipped with W transmitters. That is, each end-device can transmit simultaneously W bursts, one per wavelength. This case can be easily modeled by simply considering $N * W$ end-devices rather than N end-devices.

4. The Decomposition Algorithm

The queueing network, described in Section 3, is an *open loss queueing network*, which does not have a product form solution. However, this network possesses the Markovian property because both the duration of the bursts and their interarrival times are exponentially distributed. The underlying Markov process of this queueing network can be completely described by the tuple $(n_{12}, n_{23}, ..., n_{L-1,L})$, where n_{ij} is the number of customers in node ij and L is the total number of links. Depending on the size of the OBS path, however, the state space of this Markov process can become quite large. For example, an OBS path of 9 links, where each link's capacity is 16 wavelengths, will result in a Markov process with 140,930,306 states. For this reason, we analyze the network approximately, by decomposing it into small sub-systems. Each sub-system can be analyzed numerically because its governing process is Markovian. In order to analyze each sub-system, we need information from its adjacent sub-systems, which leads to an iterative algorithm. Below, we illustrate our decomposition algorithm through an example, and in Section 4.2 we give its general form.

4.1. An Example

In this example, we consider an OBS path which consists of 6 WDM links connected in tandem. Each fiber has two wavelengths, i.e., $W = 2$. The traffic load $\lambda_1(n_{12})$ to the OBS path is generated by $N = 5$ OBS end-devices, modeled by the multiplexed IDLE-ON process described in Section 3.2. In addition, there is a Poisson-distributed *local traffic* at each link with an average arrival rate γ.

We mode this OBS path by the five-node tandem queueing network, shown in Figure 6. The queueing network is decomposed into two sub-systems each containing three nodes. Sub-system 1 consists of nodes 12, 23 and 34 while sub-system 2 consists of nodes 34, 45 and 56. We note that the two sub-systems overlap with node 34.

The state of each sub-system can be described by a 3-tuple vector, e.g., in sub-system 1 the state description vector is (n_{12}, n_{23}, n_{34}). The state space of each sub-system is subject to the constraints (1) and (2) and it consists of the following states:

$$(000), (001), (002), (010), (011), (020), (100),$$
$$(101), (102), (110), (111), (200), (201), (202)$$

The burst loss at each link is calculated from the steady-state probability vectors of each sub-system, which we denote by π_i, where $i = 1$ or 2. The steady-state probabilities π_i are obtained numerically by solving the following linear equations in matrix form:

Event	Rate	Transition States	Conditions
cross arrival to 12	$\lambda_1(n_{12})$	$(n_{12}, n_{23}, n_{34}) \rightarrow (n_{12}+1, n_{23}, n_{34})$ no transition	if $n_{12}+n_{23} < W$ if $n_{12}+n_{23} = W$
local arrival to 23	γ	$(n_{12}, n_{23}, n_{34}) \rightarrow (n_{12}, n_{23}+1, n_{34})$ no transition	if $n_{12}+n_{23} < W$ and $n_{23}+n_{34} < W$ if $n_{12}+n_{23} = W$ or $n_{23}+n_{34} = W$
local arrival to 34	γ	$(n_{12}, n_{23}, n_{34}) \rightarrow (n_{12}, n_{23}, n_{34}+1)$ no transition	if $n_{23}+n_{34} < W$ and $n_{34}+n_{45} < W$ if $n_{23}+n_{34} = W$ or $n_{34}+n_{45} = W$
transition from 12 to 23	$n_{12}\mu$	$(n_{12}, n_{23}, n_{34}) \rightarrow (n_{12}-1, n_{23}+1, n_{34})$ $(n_{12}, n_{23}, n_{34}) \rightarrow (n_{12}-1, n_{23}, n_{34})$	if $n_{23}+n_{34} < W$ if $n_{23}+n_{34} = W$
transition from 23 to 34	$n_{23}\mu$	$(n_{12}, n_{23}, n_{34}) \rightarrow (n_{12}, n_{23}-1, n_{34}+1)$ $(n_{12}, n_{23}, n_{34}) \rightarrow (n_{12}, n_{23}-1, n_{34})$	if $n_{34}+n_{45} < W$ if $n_{34}+n_{45} = W$
departure from 34	$n_{34}\mu$	$(n_{12}, n_{23}, n_{34}) \rightarrow (n_{12}, n_{23}, n_{34}-1)$	always

Table 1. Possible State Transitions

$$\pi_i Q_i = \mathbf{0} \tag{4}$$

$$\pi_i e_i = \mathbf{1} \tag{5}$$

where (5) is the normalization condition, $e_i = (1, 1, ..., 1)^T$ and Q_i is the rate matrix of each sub-system. This system of linear equations is solved using the Gauss-Seidel method (see Stewart [11]).

Sub-system 1. Assume that at time t, sub-system 1 is in state (n_{12}, n_{23}, n_{34}). Then at time $t + \triangle t$ it may transition into one of the states shown in Table 1. The rate matrix Q_1 is generated using those state transitions. Note, however, that for some of the transitions we need n_{45} but in sub-system 1 we do not know its value. We address this problem by conditioning some of the state transitions in sub-system 1 on the state of sub-system 2. We do this by using node 34, which is common for both sub-systems 1 and 2. In sub-system 2 we find the conditional probability r that link 4 is not full given that there are exactly n_{34} bursts in node 34:

$$r(n_{34}) = Prob\{n_{45} < W - n_{34} |\, n_{34}\} \tag{6}$$

We use this probability information in sub-system 1. Instead of determining whether the equality $n_{34}+n_{45} < W$ is satisfied, we use the probability $r(n_{34})$. The successful rate of transition from node 23 to 34 becomes $r(n_{34})n_{23}\mu$. In other words, the sub-system transitions with that rate from state (n_{12}, n_{23}, n_{34}) to state $(n_{12}, n_{23}-1, n_{34}+1)$. The unsuccessful rate, i.e., transition form (n_{12}, n_{23}, n_{34}) to $(n_{12}, n_{23}-1, n_{34})$ becomes $(1 - r(n_{34}))n_{23}\mu$.

Sub-system 2. Before we can generate the rate matrix Q_2, we need to determine the arrival process to sub-system 2, which consists of the cross traffic, coming from node 23, and the local traffic at link 3. Note that the departure rate of the cross traffic from node 23 is $n_{23}\mu$, $0 \leq n_{23} \leq W$. However, in sub-system 2 we do not know the value of n_{23}. Therefore in sub-system 1, we express the departure rate from node 23 as a function of n_{34} and we use it as the *state-dependent* arrival rate to sub-system 2:

$$\lambda_2(n_{34}) = \sum_{j=0}^{W} Prob\{n_{23} = j|n_{34}\}\, j\mu,\ 0 \leq n_{34} \leq W \tag{7}$$

Now, we can generate the rate matrix Q_2 for sub-system 2 using transitions similar to sub-system 1. In this case, however, there is no need to condition any of the transitions because sub-system 2 contains all the information about the links that make its nodes.

The Iterative Algorithm. We analyze this queueing network with an iterative algorithm. In the initialization step, we ignore any of the link dependencies between the sub-systems in order to get initial guesses for the steady-state probabilities π_i, $i = 1$ or 2. In the iterative step, we first solve for π_1 in sub-system 1 by conditioning some of the transitions with the probabilities $r(n_{34})$, calculated based on the value of π_2 from the previous iteration. Next, we determine the state-dependent arrival rate λ_2 to sub-system 2 based on the current estimate of π_1 and we solve for π_2. We repeat the iterative step until the steady-state probabilities converge.

Calculation of the Burst Loss Probability. We now show how to calculate the burst loss probability at each link of the path. The burst loss probability is the probability that there are no available wavelengths upon arrival. It refers to the portion of arriving bursts at link i, which find that all the wavelengths on link i are busy. We denote the cross traffic burst loss probability at link i with b_i.

Since our arrivals are state-dependent, the burst loss probabilities are obtained based the steady-state vectors π in combination with the appropriate arrival rates. For example, the burst loss probability at link 2 is given by:

$$b_2 = \frac{\sum_{i=0}^{W} \lambda_1(i)\ Prob\{Link\ 2\ is\ full\ |n_{12} = i\}}{\sum_{i=0}^{W} \lambda_1(i)\ Prob\{n_{12} = i\}} \tag{8}$$

where the probabilities are found based on π_1. The form of the numerator is justified by the fact that λ_1 is a function of n_{12}. The denominator represents the average arrival rate.

The burst loss probability at link 3 is also found based on π_1 and it is given by:

$$b_3 = \frac{\sum_{i=0}^{W} i\mu\ Prob\{Link\ 3\ is\ full\ |n_{12} = i\}}{\sum_{i=0}^{W} i\mu\ Prob\{n_{12} = i\}} \quad (9)$$

Note that the calculation of b_3 is similar to b_2 except that we don't know the arrival rate to node 23 and instead we use the departure rate from node 12.

The remaining burst lost probabilities are calculated based on π_2. Burst loss at link 4, i.e, b_4 is calculated similarly to b_2 while b_5 is similar to b_3. We note that there is no burst loss on the first and the last links, since each of these links is dedicated to an end-device.

4.2. Generalization of the Algorithm

We now generalize our ideas to an OBS path with any number of links. First, we construct a queueing network so that each node represents the number of links that are simultaneously possessed by a burst, see Section 3. Next, we decompose this queueing network into sub-systems and we denote the number of sub-systems by s. There are different ways to decompose the queueing network but a large number of nodes per sub-system and a large number of wavelengths will result in an unmanageable number of states. Even for a sub-system with 3 nodes, the number of states is in the order of $O(W^3)$. Therefore, a decomposition of *3 nodes per sub-system with single node overlap* is a good first choice while W is not too large. Note that with this type of decomposition if the number of nodes in the queueing network is even, then the last sub-system will contain only 2 nodes. This is not a problem since the analysis of a 2-node sub-system is identical to a 3-node sub-system.

For a very large W a decomposition of *2 nodes per sub-system with single node overlap* is a better choice. The number of states for a sub-system of 2 nodes is in the order of $O(W^2)$ and therefore OBS networks with much greater number of wavelengths can be analyzed.

The generalized algorithm is outlined in Algorithm 1. Note that the superscript refers to the iteration number and the convergence condition is met when the steady-state probabilities converge, i.e, $|\pi_j^{(i)} - \pi_j^{(i-1)}| < \epsilon$, $1 \leq j \leq s$ and $\epsilon = 10^{-6}$. Once the steady-state probability vectors π_j, $1 \leq j \leq s$ are computed, the burst loss probabilities at each link can be obtained.

//Initialization Step ;
for $j = 1 : s$ **do**
  if $j = 1$ **then**
    $\lambda_j^{(1)}(n_{12}) = (N - n_{12})\alpha$;
  else
    calculate $\lambda_j^{(1)}(n_{j,j+1})$ based on $\pi_{j-1}^{(1)}$;
  //solve each sub-system independently ;
  $r_j^{(1)} = 1$;
  generate $Q_j^{(i)}$;
  solve for $\pi_j^{(i)}$;
//Iterative Step ;
for $i = 2$ *until convergence* **do**
  for $j = 1 : s$ **do**
    if $j \neq 1$ **then**
      calculate $\lambda_j^{(i)}(n_{j,j+1})$ based on $\pi_{j-1}^{(i)}$;
    //get the condition probabilities for all sub-systems but the last one ;
    if $j \neq s$ **then**
      calculate $r_j^{(i)}$ based on $\pi_{j+1}^{(i-1)}$;
    generate $Q_j^{(i)}$;
    solve for $\pi_j^{(i)}$;

Algorithm 1: Generalization for a Queueing Network with Any Number of Nodes

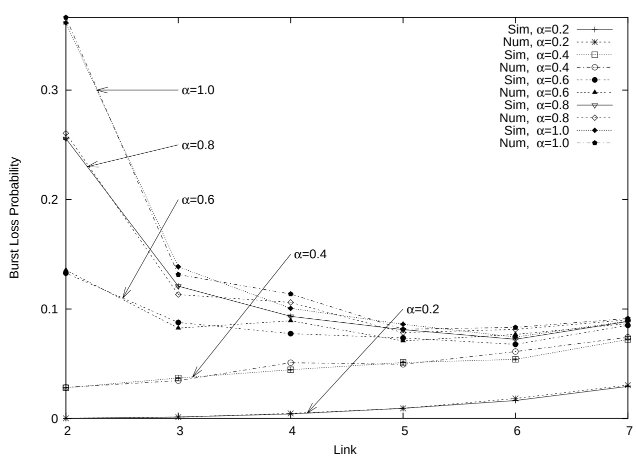

Figure 7. Burst Loss Probability for W=16, L=8, N=32, low to high load

5. Numerical Results

We now present numerical results for the *cross* traffic burst loss probability at each link of an OBS path by utilizing our decomposition algorithm and we compare them to

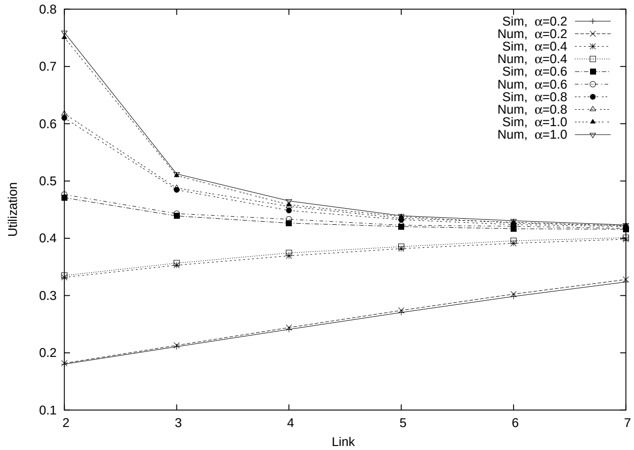

Figure 8. Utilization $\lambda/(\mu W)$ for W=16, L=8, N=32, low to high load

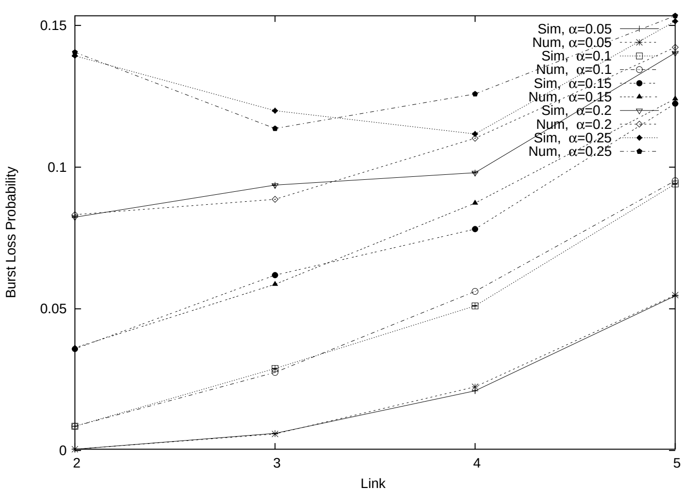

Figure 9. Burst Loss Probability for W=8, L=6, N=16, low to moderate load

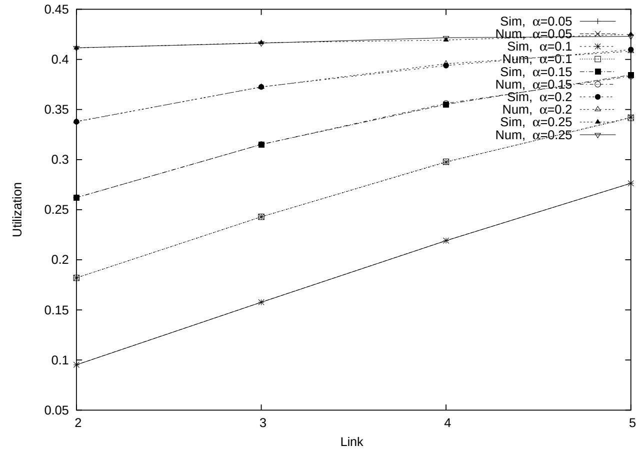

Figure 10. Utilization $\lambda/(\mu W)$ for W=8, L=6, N=16, low to moderate load

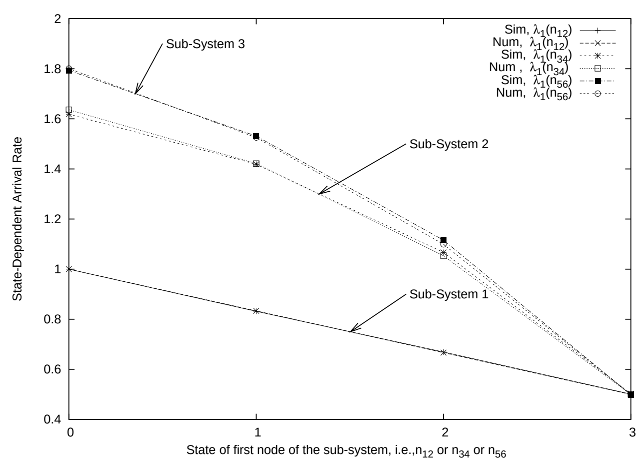

Figure 11. State-Dependent Arrival Rates for W=3, L=8, $\gamma = 0.5$, N=32 and $\alpha = 0.5$

simulation results. The analytical results are obtained using Matlab code, while the simulation results are obtained with a custom event-driven C++ simulator. For each reported statistic, we ran the simulation 30 times for a sufficiently long time in order to compute the 95% confidence intervals. Note, that the simulation results are plotted along with their confidence intervals but the confidence intervals are quite small and hardly visible on the plots. We only present a few specific examples but we tested our algorithm for a variety of input parameters and we found it to have good accuracy.

First, we present results for an OBS path of 8 links,i.e. $L = 8$, which we model as a tandem queueing network of 7 loss nodes. Each link has $W = 16$ wavelengths and there are $N = 32$ OBS transmitting end-devices, each modeled by an IDLE-ON source. The intensity of the cross traffic is varied based on the time spent in the IDLE state, i.e, based on the value of α. The local arrivals are Poisson-distributed and their average rate is the same at each link of the OBS path, i.e, $\gamma = 0.5$. The analytical results are obtained by decomposing the queueing network into 3 sub-systems each containing 3 nodes and overlapping by a single node.

In Figure 7, we plot the burst loss probability b_i at each link i, where $i = 2, 3..., 7$ for various values of α. The plot verifies that our analytical results match quite closely the results obtained through simulation. There is a slightly higher error at links 4 and 6, whose burst loss probabilities are calculated at the overlap nodes between the three sub-systems. The slightly higher error is due to the approximation used in calculating the state-dependent arrivals.

We observe a *filtering effect* of the burst loss probabilities. That is, as we increase the load at the front of the OBS path, the burst loss probability at link 2 rapidly increases but that phenomenon does not carry over to the other links in the path. Each link acts as a filter because it drops some of its incoming bursts and thus the load to the following link is reduced. In addition, there is a tendency for the burst loss probability to slightly increase from link 2 to link 7, which is caused by local arrivals at each link. The reason is that any local traffic burst, that is not lost at the link where it arrives, is carried on as part of the cross traffic to the following link. We do not plot the cross traffic burst loss at links 1 and 8 because it is zero.

For this scenario, we also plot the utilization $\lambda/(\mu W)$ of each link in Figure 8. We observe that at the lower load the link utilization increases from link 2 to link 7 because of the added local arrivals. However, as we increase the load, the utilization of links 5,6 and 7 converges. Once again, that is explained by the previously noted filtering effect. It is also interesting to note that at 50% and below utilization the burst loss probability is around 0.1.

In Figure 9 we plot the burst loss probabilities for an OBS path with $L = 6$ links and $W = 8$ wavelengths but this time the load is low to moderate. Again, the accuracy of our algorithm is very good with the largest error at link 4, which is the overlap node in the analytical solution. The utilization of this scenario is plotted in Figure 10. Note again, that at utilization below 50%, the burst loss is kept below 0.15.

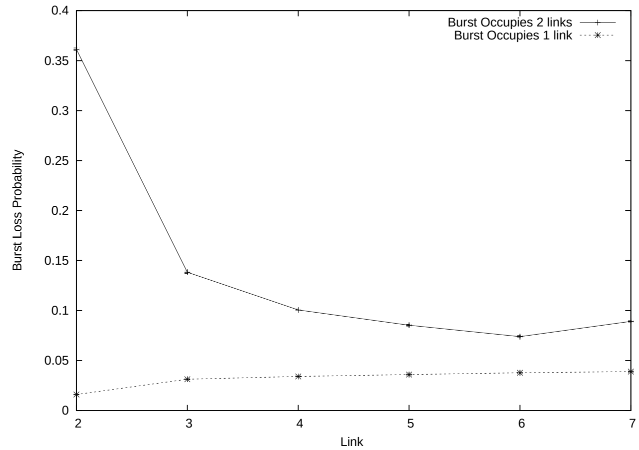

Figure 12. Burst Loss Probability for W=16, L=8, $\gamma = 0.5$,N=32 and $\alpha = 0.5$

In our algorithm, we assume state-dependent arrival rates to each sub-system. We show a validation of this assumption in Figure 11, where we have plotted the state-dependent arrival rates per sub-system. From the figure, one can see that our simulation and analytical results match very closely.

In Figure 12 we compare the simulation results for the burst loss probability in the case where bursts hold only a wavelength on a single link at a time to the case where bursts require a wavelength on two consecutive links. We keep all the other input parameters constant. We observe that the OBS path with bursts which occupy two wavelengths on two consecutive links, has a considerably higher burst loss probability because each burst requires more network resources. Therefore, in order to correctly evaluate the performance of an OBS network it is necessary to determine whether or not bursts will span multiple links.

6. Conclusions

In this paper, we considered the performance of a path in an OBS network, where bursts simultaneously possess wavelengths on two consecutive links. We analytically analyzed this OBS path by constructing an open queueing network with IDLE-ON arrival process. We developed a decomposition algorithm and we calculated the end-to-end burst loss probabilities at each link of the OBS path. The accuracy of our algorithm was verified by simulation. We found a *filtering effect* on the traffic due to the burst losses at each link, which can not be accurately captured in studies of a single OBS node. We also found that at 50% or below link utilization the burst loss probability is only about 0.1, however, that loss probability rapidly increases as the utilization increases. Finally, we found that the burst loss probability is much higher in the case where bursts occupy two links at a time to the case where each burst possess a wavelength on single link. We are currently working on extending our algorithm to an OBS network with a mix of bursts that span a single link and bursts that span multiple links.

References

[1] Y. Chen, H. Wu, D. Xu, and C. Qiao. Performance analysis of optical burst switched node with deflection routing. In *Proceedings of the IEEE International Conference on Communications, 2003. ICC '03.*, volume 2, pages 1355 –1359, May 2003.

[2] A. Detti, V. Eramo, and M. Listanti. Performance evaluation of a new technique for IP support in a WDM Optical Network: Optical Composite Burst Switching (OCBS). *IEEE Journal of Lightwave Technology*, 20(2):154–165, February 2002.

[3] K. Dolzer, C. Gauger, J. Spath, and S. Bodamer. Evaluation of reservation mechanisms for optical burst switching. *AEU International Journal of Electronics and Communications*, 55(1), 2001. January.

[4] V. Frost and B. Melamed. Traffi c modeling for telecommunications networks. *IEEE Communications Magazine*, 32(3):70–81, March 1994.

[5] C.-F. Hsu, T.-L. Liu, and N.-F. Huang. On the deflection routing in QoS supported optical burst-switched networks. *IEEE International Conference on Communications*, 5:2786–2790, 2002.

[6] X. Lu and B. Mark. A new performance model of optical burst switching with fi ber delay lines. In *Proceedings of the IEEE International Conference on Communications, 2003. ICC '03.*, volume 2, pages 1365 –1369, May 2003.

[7] N. Madamopoulos, M. D. Vaughn, L. Nederlof, and R. E. Wagner. Study of a U.S. metropolitan network: scenarios, equipment, requirements, and implications for the carriers. *Optical Networks Magazine*, 4(6):90–106, 2003.

[8] M. Neuts, Z. Rosberg, H. L. Vu, J. White, and M. Zukerman. Performance analysis of optical composite burst switching. *IEEE Communications Letters*, 6(8):346– 348, August 2002.

[9] C. Qiao and M. Yoo. "Optical burst switching (OBS)- a new paradigm for an Optical Internet". *Journal of High Speed Networks*, 8(1):69–84, January 1999.

[10] Z. Rosberg, H. L. Vu, M. Zukerman, and J. White. Performance analyses of optical burst-switching networks. *IEEE Journal on Selected Areas in Communications*, 21(7), Sept. 2003.

[11] W. J. Stewart. *Introduction to the Numerical Solution of Markov Chains*. Princeton University Press, 1994.

[12] H. L. Vu and M. Zukerman. Blocking probability for priority classes in optical burst switching networks. *IEEE Communications Letters*, 6(5), May 2002.

[13] J. Wei and R. McFarland. Just-in-time signaling for WDM optical burst switching networks. *Journal of Lightwave Technology*, 18(12):2019 –2037, December 2000.

[14] L. Xu, H. Perros, and G. Rouskas. Performance analysis of an edge optical burst switching node with a large number of wavelengths. In *Proceeding of the 18th International Teletraffi c Congress (ITC-18)*, pages 891–900, Berlin, Germany, August 31-September 5 2003.

[15] L. Xu, H. Perros, and G. Rouskas. A queueing network model of an edge optical burst switching node. In *Proceedings of IEEE Infocom 2003*, pages 2019–2029, San Francisco, CA, April 2003.

[16] M. Yoo, C. Qiao, and S. Dixit. QoS performance of optical burst switching in IP-over-WDM networks selected areas in communications. *IEEE Journal on Areas in Communications*, 18(10):2062–2071, 2000. October.