An Analytical Study of Optical Burst Switching Aggregation Strategies

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Abstract

Optical Burst Switching (OBS) is considered as a promising switching technique for the next generation of optical networks. A solid understanding of the characteristics of OBS traffic is the prerequisite to efficiently address a series of problems such as network dimensioning, performance optimization, routing and load balancing. In this paper we address this issue presenting the first exact and complete analytical model of OBS traffic with Poissonian IP input traffic. We demonstrate that with the standard aggregation strategies this traffic is not Poisson. In addition we present a new aggregation strategy that definitely generates poisson traffic, which enables the use of all published results which had assumed poisson traffic. Our analytical model includes the exact distributions of the burst interarrival time, burst size, burst formation time and number of IP packets per burst, together with tractable expressions for their mean and variance.

1. Introduction

Optical burst switching (OBS) is a promising solution for all-optical wavelength-division multiplexing (WDM) networks. It offers to some extent the flexibility and efficient bandwidth usage of optical packet switching networks, while taking into account the limitations of the current all-optical technology. For this reason it can be seen as an intermediate evolutionary step between all-optical wavelength routing networks and optical packet switching networks [1].

An OBS network (see fig. 1) is basically a packet switched network with large packets called bursts. Packet data units of higher layers (e.g. IP packets) are collected at the edge nodes of the network and sorted according to their destination (and possibly QoS class). Incoming packets with the same destination (and QoS requirements) are grouped together in a buffer until a certain *aggregation strategy* decides to stop the process. This block of collected packets, all with the same destination (and QoS class), is called a burst. Prior to the transmission of a burst, a control packet is created and immediately sent through a signaling path to the burst destination in order to configure the optical light path for the burst transmission. After waiting an offset time, the burst itself is sent through this path. Standard OBS networks do not use ACK packets in order to acknowledge the establishment of an end-to-end path, and for this reason collisions among optical bursts might take place inside the optical network.

The aggregation strategy in the edge nodes notably influences the traffic characteristics in an OBS network, since it determines among others the burst size distribution, the burst interarrival time distribution, the burst formation time distribution and the distribution of the number of higher layer packets per burst. At the same time network traffic characteristics have a great impact on the whole network performance in terms of blocking probability, throughput and delay. For this reason a close study on the different aggregation strategies which have been proposed in the literature is vital for understanding the behavior of OBS networks in terms of performance. The best approach is to study them by means of a complete and exact analytical model. The motivation for the work presented in this paper is to provide such model in order to be able to fully understand and describe the basic functioning of OBS networks.

Some analytical models have been described in the literature with the purpose of studying the traffic statistics of assembled burst traffic in OBS networks using different aggregation strategies [2][3][4][5]. However, all of these models use simplifications of some kind in order to obtain tractable mathematical expressions. For instance [2] approximates the burst arrival process with a three-state Markov process that alternates between idle times and the transmission of short or long bursts. In [3] a more accurate analytical description of the burst formation process is provided. Under the Poissonian assumption the distribution for the burst interarrival time is provided for the cases of constant and exponentially distributed IP packet size. Finally, the limiting behavior of the burst size distribution as the number of collected IP packets per burst increases is studied.

In [4] the impact of self-similarity on the assembled traffic characteristics is studied.

One of the main possible applications of an accurate burst traffic characterization is the use of adequate analytical models in order to evaluate OBS performance [6][7][8][9][10].

We present a complete analytical model for three of the most important aggregation strategies which have been described in the literature, and in addition we introduce a new aggregation strategy with its correspondent analytical model. In our model we consider a general IP packet size distribution and therefore our results regarding the burst interarrival time distribution can be considered as a generalization of those presented in [3]. In addition to the burst interarrival time distribution we obtain new results regarding the exact distributions for the burst size, burst formation time and number of higher layer packets per burst, as well as their respective moment generating functions (MGF). With the help of the MGF we provide a table with the mean and variance of the most important distributions in each aggregation strategy.

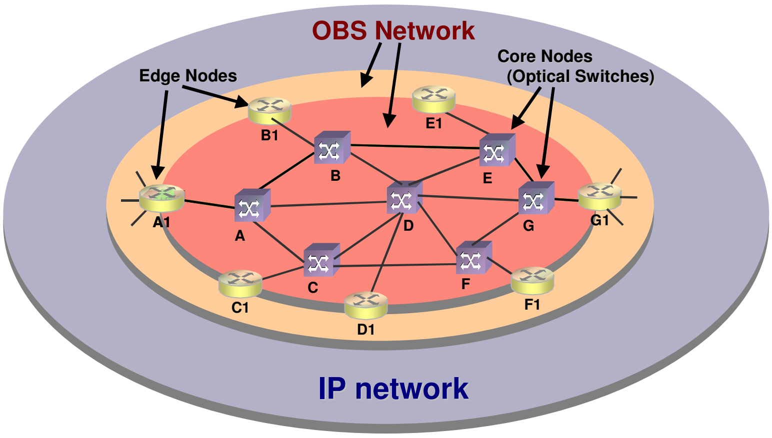

Figure 1. Simple model of an OBS network

The paper is divided as follows. Section 2 presents the analytical models of the different aggregation strategies in the edge node. Section 3 validates the analytical results obtained in the previous section with the help of simulations. Section 4 compares the different burst aggregation mechanisms and describes some implications. Finally, section 5 concludes this paper with some final considerations and remarks.

2. Burst aggregation strategies in the edge mode

The burst aggregation strategy in an OBS network is extremely important. It determines the traffic characteristics and therefore influences the whole OBS network performance, like for instance the blocking probability, the packet delay, or the throughput. This section provides a novel complete analytical study of some of the most important aggregation strategies together with a new one: the aggregation strategy with random selection. The study is complete in the sense that no simplification of the aggregation strategies has been made and that no approximation of the calculations has been used. It is valid in core and high-loaded metro networks.

Most OBS networks are foreseen to be deployed in the Internet, so throughout the text higher layer packets will be assumed to be IP packets. Measurements [11] and analytical studies [12] show that Internet traffic tends to resemble a pure Poisson process as the number of multiplexed sources increases. A recent empirical study of highly aggregated data traces [13] concludes that below the scale of seconds the interarrivals of the packet arrival process are independent and exponentially distributed, which univocally points towards a Poisson arrival process. The burst formation time in OBS networks normally is below the second-scale and therefore it will be assumed in some sections of this paper that the IP packet arrival process is Poisson. This assumption sets the focus of the correspondent sections on OBS networks where the IP traffic aggregation level is high enough in order for the conclusions in [13] to hold. Normally this is the case in core and possibly in high-loaded metro OBS networks. Some other results like the distribution of the number of IP packets per burst and of the burst size in the case of the burst aggregation strategy with buffer limit do not make any assumption regarding the statistical nature of the IP packet arrival process.

The analytical models presented in this section are valid for any IP packet size distribution ***S***. In particular, ***S*** could follow the tri-modal distribution that according to measurements [14] serves as a good approximation of the IP packet size distribution.

2.1. Burst aggregation strategy with timeouts

In this strategy the burst formation time is constant and equal to the aggregation time ***T***. Each burst aggregation queue is equipped with a time counter. When a packet arrives at an empty burst aggregation queue, the time counter is started with t = 0. Further incoming IP packets are collected in the burst aggregation queue until the time counter reaches the value t = ***T***. Then a burst is created with the contents of the burst aggregation queue, and it is queued for transmission on the data channel. The time counter is reset to zero and remains so until the next packet arrives to the queue. The aggregation strategy must guarantee that no segmentation of IP packets takes place when the timeout is triggered before an IP packet has completely arrived at the aggregation buffer.

2.1.1. Interarrival time between bursts and burst formation time distributions.

The arrival rate of the IP packets is determined by a Poisson process of rate λ packets/second. Therefore, the interarrival time between IP packets E is negative-exponentially distributed with parameter λ. The interarrival time between bursts Z is defined as the time elapsed between two consecutive burst arrivals, as shown at the bottom of figure 2:

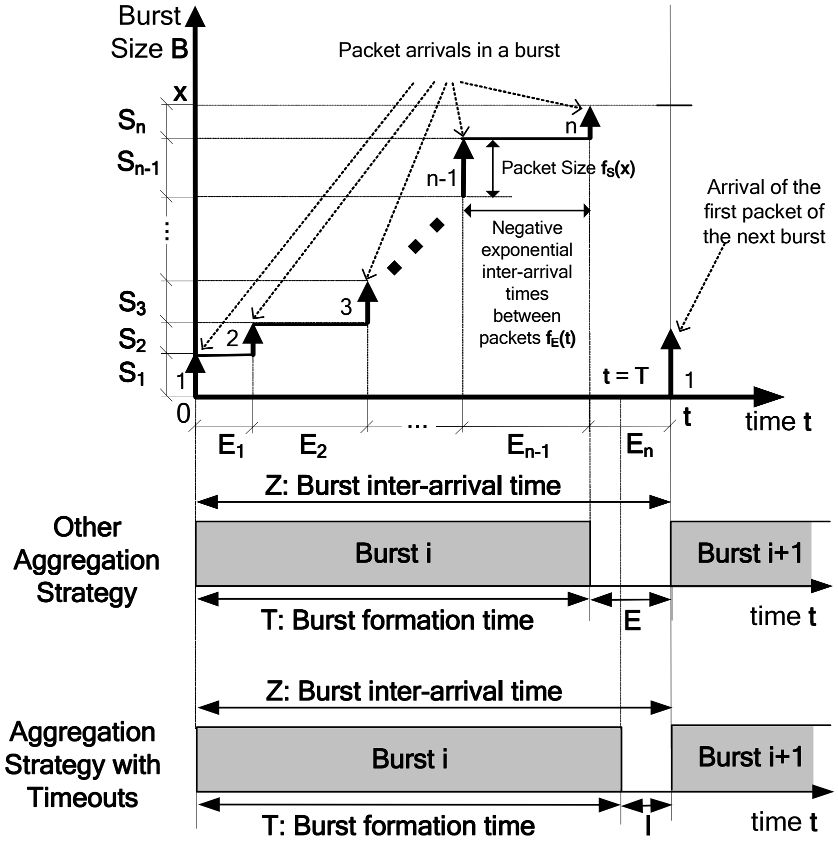

Figure 2. Relationship among the most important variables in an edge node. E is the interarrival time between packets, and I is the time to the next burst arrival.

The formation time of a burst is the same for all bursts and equal to the (constant) aggregation time ***T*** of the burst aggregation strategy. Therefore it can be modeled with a constant random variable (r.v.) ***T***.

According to the aggregation strategy, the time to the next burst arrival ***I*** is the time to the next IP packet arrival measured from time t = ***T*** (see bottom of figure 2). Due to the memoryless property of a Poisson process, the time to the next packet arrival is equal to the interarrival time between packets ***E*** independently from the reference time t = ***T***, and therefore ***I*** is also exponential with parameter λ ($I = E$). The interarrival time between bursts ***Z*** is $Z = T + I$ (see figure 2). Therefore, the PDF of the interarrival time between bursts ***Z*** is an exponential distribution shifted ***T*** seconds to the right:

$$f_Z(t) = f_I(t-T) = \lambda \cdot e^{-\lambda \cdot (t-T)} \qquad \textbf{(1)}$$

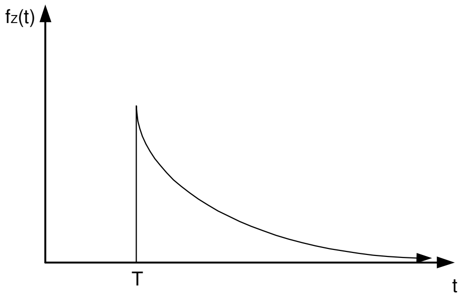

Figure 3. Burst inter-arrival time distribution

The MGF (Moment Generating Function) of the interarrival time between bursts is:

$$\Phi_Z(s) = e^{sT} \cdot \Phi_I(s) = e^{sT} \cdot \frac{\lambda}{\lambda - s} \qquad \textbf{(2)}$$

With the MGF the n-th moment of the interarrival time between bursts can be easily calculated by evaluating the n-th derivative of $\Phi_Z(s)$ in $s = 0$. For instance, the average interarrival time between bursts can be calculated as:

$$E[Z] = \frac{\partial \Phi_Z(s)}{\partial s}\Big|_{s=0} = T + \frac{1}{\lambda}.$$

2.1.2. Distributions of burst size and number of IP packets per burst.

The time counter begins with the arrival of the first IP packet of a burst at the edge node, i.e. in t=0 there is always one IP packet in the buffer. Subsequent IP packets are collected in the buffer in order to form a burst. After *T* seconds the aggregation process is finished and the burst is ready. The amount of bits arriving at the edge node in *T* seconds ***X(T)*** is a random sum of random variables. In particular, if the r.v. $S^{(i)}$ represents the size of IP packet i, $X(T) = S^{(1)} + \dots + S^{(K)}$, where the r.v. ***K*** represents the number IP packets arriving at the edge node in *T* seconds and $S^{(i)}$ are independent and identically distributed (*iid*) r.v.. The burst size ***B*** is equal to the amount of bits arriving at the edge node in *T* seconds ***X(T)*** plus the size $S^{(i)}$ of the first IP packet arriving at t=0, that is $B = S^{(1)} + \dots + S^{(N)}$, where the r.v. $N = K + 1$ represents the number of IP packets per burst. Figure 2 (top of the figure) illustrates the burst generation process graphically.

The r.v. ***K*** is a Poisson r.v. corresponding to the number of arrivals during *T* seconds in a Poisson counting process ***K(t)*** of parameter λ.. Therefore the r.v. ***N*** is the number of arrivals of the counting process ***K(t)*** conditioned to the fact that in t=0 an IP packet has arrived, of which probability mass function (PMF) is given by:

$$P_N(n) = \begin{cases} \dfrac{(\lambda \cdot T)^{n-1} e^{-\lambda \cdot T}}{(n-1)!} & n = 1,2,\dots \\ 0 & Otherwise \end{cases} \qquad \textbf{(3)}$$

where $P_N(n)$ represents the probability that a burst has *n* IP packets.

The PDF of ***B*** is the PDF of a random sum of random variables:

$$f_B(x) = \sum_{n=1}^{\infty} P_N(n) f_{S_n}(x) \qquad \textbf{(4)}$$

where $P_N(n)$ is given by equation 3, $f_S(x)$ is the PDF of ***S*** and $f_{S_n}(x)$ is the convolution of $f_S(x)$ with itself *n* times. It can be observed in equation 4, that the burst size depends on λ and *T* only through their product λ*T, this fact can be exploited in order to set the parameter *T* in the edge node depending on the traffic load conditions. Moreover, the value λ*T+1 represents the average number of IP packets per burst.

It may be difficult to simplify equation 4 for some packet size distributions since the direct convolution of $f_S(x)$ with itself *n* times is not always straightforward. Therefore, it is very useful to obtain the MGF of the burst size distribution. Taking into account that $B = S^{(1)} + \dots + S^{(K)}$ is a random sum of $N = K + 1$ iid random variables where the r.v. ***K*** is Poisson distributed [15], the MGF is:

$$\Phi_B(s) = \Phi_S(s) \cdot \Phi_N\left(Ln\Phi_S(s)\right) = \Phi_S(s) \cdot e^{\lambda \cdot T(\Phi_S(s)-1)} \qquad \textbf{(5)}$$

2.2. Burst aggregation strategy with buffer limit

Incoming IP packets with the same destination edge node (and optionally QoS class) are collected in a buffer. Whenever the buffer occupation level L exceeds a certain limit B_{max} *(i.e.* $L > B_{max}$*)*, the buffer contents are queued for transmission on the data channel. According to this the maximum burst size will be $B_{max} + S_{max}$, where S_{max} is the maximum IP packet size, and the minimum burst size B_{max} + *1 Byte*.

2.2.1. Distribution of the number of IP packets per burst.

In the burst aggregation strategy with timeouts, the probability that a burst has *n* packets is Poisson distributed (conditioned to one packet arrival in t=0). In this case the probability that a burst has *n* packets is the probability that *n-1* IP packets don't overflow the buffer size B_{max} and *n* IP packets do: $P_N(n) = Pr(S_{n-1} \leq B_{max} < S_n)$. This can be expressed (using the law of total probability and the fact that the r.v. $\boldsymbol{S_i}$ are independent) as:

$$
\begin{aligned}
&P_N(n) = \\
&\int_{b=0}^{B_{\max}} \Pr\left(S^{(n)} > B_{\max} \middle| S^{(n-1)} = b\right) \cdot f_{S_{n-1}}(b) \cdot db = \\
&\int_{b=0}^{B_{\max}} \left[1 - F_S\left(B_{\max} - b\right)\right] \cdot f_{S_{n-1}}(b) \cdot db; \quad n \geq 1
\end{aligned}
\qquad (\mathbf{6})
$$

where $F_S(x)$ is the cumulative distribution function (CDF) of the r.v. $\boldsymbol{S}$. The r.v. $\boldsymbol{N}$ will be used in order to calculate the burst interarrival time and the burst size distributions. A further simplification of equation 6 depends on the particular distribution of $\boldsymbol{S}$. For instance, if $\boldsymbol{S}$ is exponentially distributed with parameter $\boldsymbol{\mu}$, equation 6 represents the distribution of the conditioned Poisson r.v. described in equation 3 but with parameter $\boldsymbol{B_{max} / \mu}$. Equation 6 is also valid for other arrival processes than Poisson.

2.2.2. Interarrival time between bursts and burst formation time distributions.

According to figure 2 the r.v. describing the burst formation time is $\boldsymbol{T} = \boldsymbol{E}^{(1)} + ... + \boldsymbol{E}^{(N-1)}$, where $\boldsymbol{E}$ is the exponential r.v. representing the interarrival times between IP packets, and the r.v. $\boldsymbol{N}$ represents the number of IP packets per burst (equation 6). Therefore, its PDF is:

$$
\begin{aligned}
&f_T(t) = \sum_{n=1}^{\infty} \Pr\left(T = t \middle| N = n\right) \cdot P_N(n) = \\
&\sum_{n=1}^{\infty} f_{E_{n-1}}(t) \cdot P_N(n)
\end{aligned}
\qquad (7)
$$

where $f_{En-1}(t)$ is the Erlang distribution with parameters λ and *n-1*.The r.v. describing the interarrival times between bursts is $\boldsymbol{Z} = \boldsymbol{T} + \boldsymbol{E}$ (see figure 2)*,* where $\boldsymbol{E}$ is exponential and $\boldsymbol{T}$ is described by equation 7. Therefore its PDF is:

$$
f_Z(t) = \sum_{n=1}^{\infty} f_{E_n}(t) \cdot P_N(n) \qquad (\mathbf{8})
$$

where $f_{En}(t)$ is the Erlang distribution with parameters λ and *n* and $P_N(n)$ is given by equation 6. Its MGF is similar to equation 5:

$$
\begin{aligned}
&\Phi_Z(s) = \Phi_N\left(Ln\Phi_E(s)\right) = \\
&\Phi_N\left[Ln\left(\frac{\lambda}{\lambda - s}\right)\right]
\end{aligned}
\qquad (\mathbf{9})
$$

2.2.3. Burst size distribution.

The number of IP packets per burst $\boldsymbol{N}$ and the IP packet size $\boldsymbol{S}$ depend on one another through equation 6. Therefore the burst size $\boldsymbol{B}$ can not be expressed as the sum of $\boldsymbol{N}$ times an IP packet size $\boldsymbol{S}$ since both r.v. are not independent. In this case we must consider the joint PDF of the r.v. $\boldsymbol{B}$ and $\boldsymbol{N}$. The probability that the burst size $\boldsymbol{B}$ equals some value $\boldsymbol{x}$ and that the number of packets per burst $\boldsymbol{N}$ equals some value $\boldsymbol{n}$ can be calculated

- using the fact that the r.v. $\boldsymbol{S}^{(i)}$ are independent, and
- under the assumption that the buffer occupancy level is $\boldsymbol{b}$ after the arrival of $\boldsymbol{n-1}$ IP packets:

$$
\begin{aligned}
&f_{B,N}(x,n) = \\
&\int_0^{B_{\max}} \Pr\left(S_n = x \middle| S_{n-1} = b\right) \cdot \Pr\left(S_{n-1} = b\right) \cdot db = \\
&\int_0^{B_{\max}} f_S(x-b) \cdot f_{S_{n-1}}(b) \cdot db; \quad x > B_{\max}, \quad n \geq 1
\end{aligned}
\qquad (\mathbf{10})
$$

Finally, the burst size distribution can be calculated as a marginal distribution of equation 10, taking into account that according to the definition of the aggregation strategy the size of a burst must be greater than the buffer size $\boldsymbol{B_{max}}$:

$$
f_B(x) = \begin{cases} \displaystyle\sum_{n=1}^{\infty} \int_0^{B_{\max}} f_S(x-b) \cdot f_{S_{n-1}}(b) \cdot db & x > B_{\max} \\ 0 & x \leq B_{\max} \end{cases} \qquad (\mathbf{11})
$$

2.3. Burst aggregation strategy with packet count limit

Incoming IP packets with the same destination edge node (and optionally QoS class) are collected in a buffer. Whenever a buffer receives *n* IP packets, its contents are queued for transmission on the data channel. According to this, the number of IP packets per burst is constant and equal to *n*.

2.3.1. Distributions of interarrival time between bursts and burst formation time.

Consider the time diagram in figure 2. The generation time of a burst ***T*** is the time elapsed while receiving *n* IP packets. Assuming negative exponential interarrival times between packets, the burst formation time can be modeled with an Erlang distribution of parameters λ and *n-1*.

As it was discussed in section 2.A.1., the time to the next burst arrival ***I*** is the interarrival time between IP packets ***E*** which is an exponential distribution of parameter λ. According to figure 2, ***Z = T + E*** which is an Erlang distribution of parameters λ and *n-1* plus an exponential distribution of parameter λ, which leads to an Erlang distribution of parameters λ and *n* [15].

$$f_Z(t) = \begin{cases} \dfrac{\lambda^n \cdot t^{n-1} \cdot e^{-\lambda \cdot t}}{(n-1)!} & t \geq 0, n = 1,2,... \\ 0 & otherwise \end{cases} \quad \textbf{(12)}$$

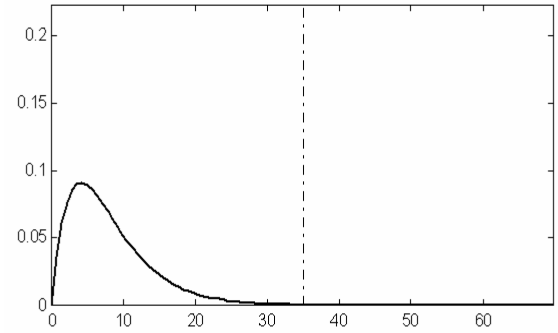

Figure 4. Shape of the burst inter-arrival time distribution

Since this distribution is well known, its MGF will not be repeated here.

2.3.2. Burst size distribution.

In this case, all the bursts have a constant number of *n* IP packets. Their size is given by a random variable which is the addition of *n* independent random variables ***S***, where ***S*** is the random variable that describes the IP packet size. Its PDF is the convolution of $f_S(x)$ with itself *n* times:

$$f_B(x) = f_{S_n}(x) \quad \textbf{(13)}$$

of which MGF is:

$$\Phi_B(s) = \left(\Phi_S(s)\right)^n \quad \textbf{(14)}$$

2.4. Burst aggregation strategy with random selection

In the literature it has been often assumed that the burst arrival process is Poisson [8][9][10]. Under this assumption, the Erlang B formula has been used in order to calculate blocking probabilities in OBS networks and in order to provide QoS guarantees for different service classes.

The results from the former sections are both good and bad news. On one side we finally know the exact form for the distribution of many interesting measures such as the burst size or burst interarrival time. On the other hand these results formally prove that the burst arrival process is definitely not Poisson since the interarrival times between bursts are not independently exponentially distributed. Therefore strictly speaking the prerequisites for the use of the Erlang B formula are not fulfilled, which introduces complexity in the analysis of the blocking probabilities. Following the philosophy in [9], the use of the Erlang B formula should be carefully studied in each aggregation strategy since now we do not have the guarantees that it will provide good approximations to the results. A simple example justifies this. The interarrival time distribution for the aggregation strategy with timeouts follows a shifted-exponential distribution (equation 1). It resembles a constant distribution (see figure 7.a) - since the random part ***I*** (see figure 2) is in practice between two and three orders of magnitude smaller than the constant part ***T*** (in fact, according to table 1, the average value 1/λ of the random component ***I*** is n-1 times smaller than ***T***, where n is the average number of IP packets per burst). For this reason, if several such edge nodes begin sending bursts at approximately the same time (or at completely different times), they will continue doing so leading to a considerably higher (lower) blocking probability than expected from the Erlang B formula, especially for low (high) network loads.

This section presents a novel burst aggregation strategy of which main feature is that it leads to a pure Poisson distributed burst arrival process, and therefore allows the use of the Erlang B formula and of the many useful results described in the literature [8][9][10].

Assuming that the IP packet arrival process is Poisson distributed, the idea of this aggregation strategy is to consider the *random selection property* of any Poisson process. This property states that if a random selection is made from a Poisson process with intensity λ such that each arrival is selected with probability p, independently of the others, the resulting process is a Poisson process with intensity $p*\lambda$. This lower-rate Poisson process ($p<1$) will mark the beginning of the optical bursts, and this allows to assure that the burst arrival process is Poisson.

Regarding the implementation, the edge node needs a simple Bernoulli random generator and a buffer to accumulate the bursts. The edge node uses the Bernoulli random generator to obtain a sequence of 1's and 0's with a certain probability p for the 1's and $1-p$ for the 0's. Each number (1 or 0) is associated to an incoming IP packet. The number 1 indicates the beginning of a new burst, while the number 0 means that the IP packet has to be padded to the burst which is being generated in the buffer. The burst formation algorithm is then very simple and it is described below:

- Every time the edge node receives an IP packet it sends it to the buffer. Then it reads the associated random number corresponding to the *next* IP packet.
- If the associated random number for the next IP packet is 1, the burst in the buffer is sent. Otherwise, do nothing.

Finally, the parameter p of the Bernoulli random generator has a very intuitive meaning. It represents the inverse of the average number of IP packets per burst $n = 1/p$.

2.4.1. Distribution of the number of IP packets per burst.

An IP packet has an associated random number of 1 with a probability p. The probability that n packet arrivals have an associated random number of 0 is $(1-p)^n$. Therefore, the probability that a burst is composed by n IP packets $P(n)$ is the probability that the first packet has an associated random number of 1 and that $n-1$ packet arrivals have an associated random number of 0. That is the probability that a burst has n IP packets is:

$$P(n) = p \cdot (1-p)^{n-1} \qquad \textbf{(15)}$$

which corresponds to a geometric distribution of parameter p.

2.4.2. Distributions of interarrival time between bursts and burst formation time.

The burst formation time begins when the first IP packet of a burst arrives (figure 2). If a burst has only one IP packet, the burst formation time is 0. If a burst has n IP packets, the burst formation time is the sum of $n-1$ exponential random variables of parameter λ, which is an Erlang random variable of parameters $n-1$ and λ. According to this:

$$f_T(t) = p \cdot \delta(t) + \sum_{n=2}^{\infty} P(n) \cdot f_{E_{n-1}}(t) \qquad \textbf{(16)}$$

where $\delta(t)$ is the delta function centered in the origin, $P(n)$ is geometrically distributed, and $f_{E_{n-1}}(t)$ is the PDF of the Erlang random variable. Simplifying equation 16:

$$f_T(t) = \begin{cases} p \cdot \delta(t) + (1-p) \cdot \lambda \cdot p \cdot e^{-\lambda \cdot p \cdot t} & t \geq 0 \\ 0 & otherwise \end{cases} \qquad \textbf{(17)}$$

That is, the burst formation time is an exponential random variable of parameter $p*\lambda$ with probability $(1-p)$ and a delta centered in the origin ($t=0$) with probability p.

The inter-arrival time between bursts **Z** is defined as the time elapsed between two consecutive burst arrivals. Every packet with an associated random number 1 marks the beginning of a burst. According to the random selection property of a Poisson process, the random process formed by those IP packets with an associated random number 1 is Poisson as well. Therefore, the inter-arrival time between those packets is negative exponentially distributed, and so it is the burst inter-arrival time.

The PDF of the inter-arrival time between bursts **Z** is:

$$f_Z(t) = \frac{\lambda}{n} \cdot e^{\frac{-\lambda \cdot t}{n}} \qquad \textbf{(17)}$$

where $n = 1/p$.

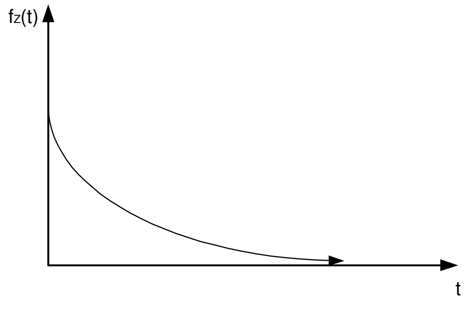

Figure 5. Burst inter-arrival time distribution is negative exponential

2.4.3. Burst size distribution.

Analogously to the derivation of equation 4, the PDF of the burst size distribution ***B*** can be expressed as follows:

$$f_B(x)=\sum_{n=1}^{\infty}P(n)\cdot f_{S_n}(x) \tag{18}$$

where $P(n)$ is geometrically distributed and $f_{sn}(x)$ is given by the PDF of the sum of a number of n iid (independent and identically distributed) IP packet size S r.v.: $\boldsymbol{S_n} = \boldsymbol{S^{(1)}} + \ldots + \boldsymbol{S^{(n)}}$. The MGF of the burst size distribution is the MGF of the random sum of ***N*** independent random variables ***S.***

$$\Phi_B(s)=\Phi_N\left(Ln\Phi_S(s)\right)=\frac{p\cdot\Phi_S(s)}{1-(1-p)\cdot\Phi_S(s)}$$

3. Numerical results

The analytical models were simulated with the help of Matlab in order to validate them. This section presents the simulation results compared with the predictions from the analytical models. In particular, the cumulative distributed functions CDF of several distributions of all aggregation strategies were compared, since the validation of the CDF implies the validation of the PDF and therefore of all the information that can be obtained from it (e.g. mean, variance and autocorrelation).

According to traffic measurements [14], the IP packet size distribution can be closely approximated with a tri-modal distribution. This distribution consists on three delta functions centered in 40, 576 and 1500 bytes with probabilities 0.6, 0.17 and 0.23 respectively. In our simulations we used this distribution with an average IP packet arrival rate of $\lambda = 1$ packet / µs. Finally, an average number of IP packets per burst equal to 101 was chosen. This determines the parameters in each aggregation strategy univocally (e.g. T, B_{max}, n and p).

Figure 7 shows the CDF's of the burst size distribution and interarrival time distribution between bursts for each aggregation strategy.

As it can be observed from figure 7, the results from the analytical model accurately match the simulation results. The variance of the burst interarrival time for the aggregation strategy with timeouts is notably lower than with the rest of the aggregation strategies. This result will be quantified in table 1. The CDF in figure 7.g. shows that the burst interarrival time distribution of the aggregation strategy with random selection is exponential. Therefore this aggregation strategy enables the use of the M/G/x model[1], which is one of the easiest, most tractable, and best understood models from the queuing theory.

The PDFs of the number of IP packets per burst in the aggregation strategies with timeouts, packet count and random selection are poisson distributed, constant and geometrically distributed respectively. These expressions will not be validated in this section, since they are well-known elements of probabilistic theory. However the PDF of the number of IP packets per burst in the aggregation strategy with buffer limit is not a well-known expression and it deserves to be taken a closer look. The following figure 6 shows the CDF of the number of IP packets per burst for the aggregation strategy with buffer limit calculated by numerical evaluation of equation 6 and by means of a simulation. The simulation parameters were the same as the ones chosen for the simulations in figure 7.

As it can be observed from figure 6, equation 6 follows the simulated results very accurately. Consequently, the information that can be extracted from the CDF (and PDF) such as its maximum, minimum, average, variance and autocorrelation will be accurate as well.

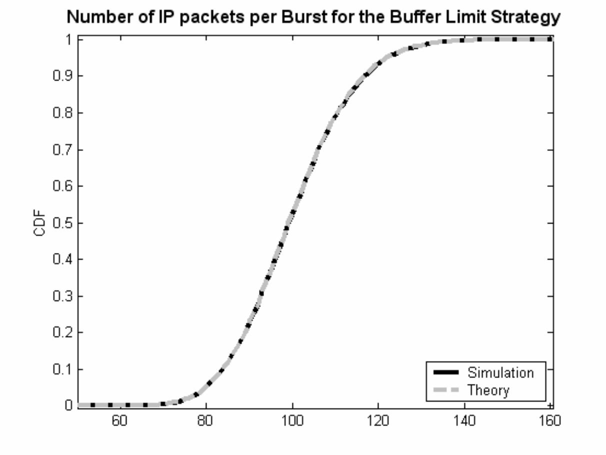

Figure 6. Comparison of the simulation vs. theoretical results for the cumulative distributed function of the number of IP packets per burst in the aggregation strategy with buffer limit

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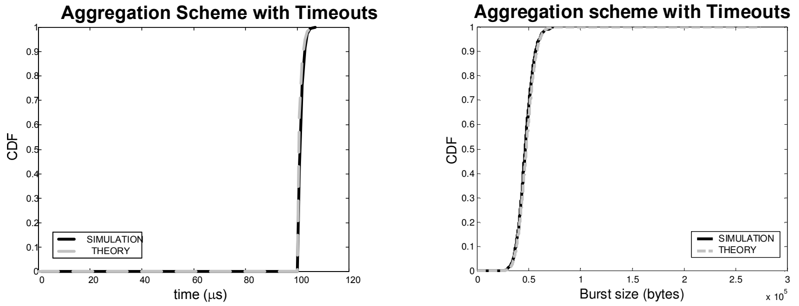
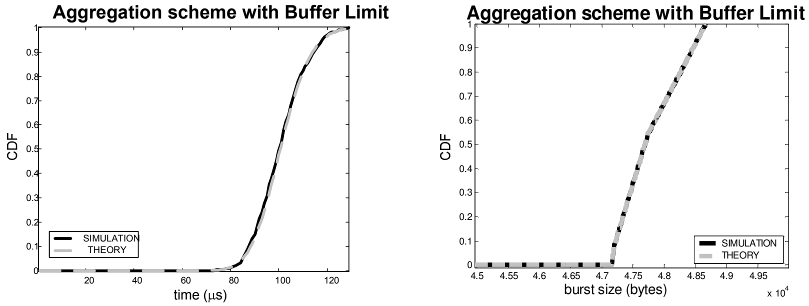
[1] In the Kendall notation in an A/B/x model A represents the interarrival time distribution, B the service time distribution and x may represent the number of servers, their storing capabilities and the service discipline. The symbol M stands for the exponential distribution and G for a general distribution.



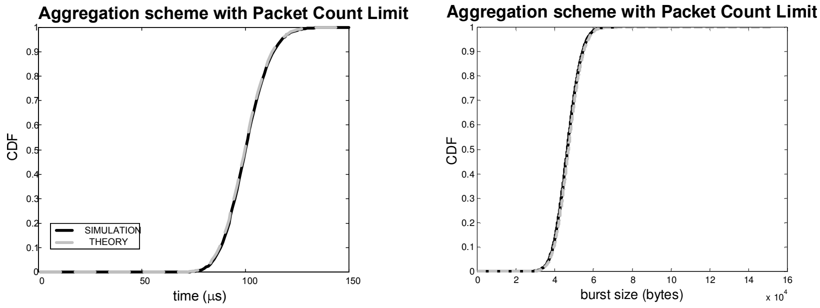
a) CDF of the Burst Inter-arrival time distribution

b) CDF of Burst size distribution



c) CDF of the Burst Inter-arrival time distribution

d) CDF of Burst size distribution



e) CDF of the Burst Inter-arrival time distribution

f) CDF of Burst size distribution

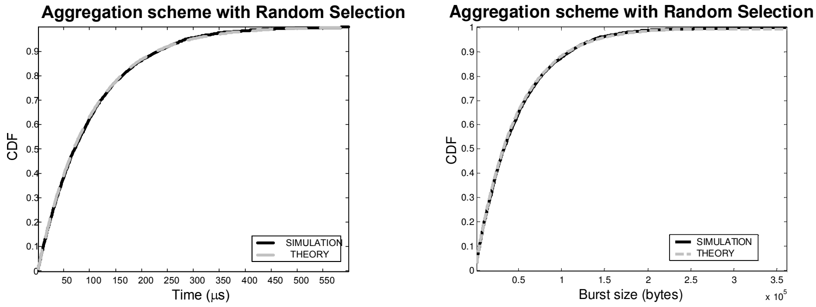


Figure 7: Analytical vs. simulative cumulative distributed functions (CDF) of the burst interarrival time and burst size distributions. A tri-modal IP packet size distribution was used, and an average number of n = 101 IP packets per burst.

4. Comparison and implications of the aggregation strategies

In section 3 the probability distributions presented in section 2 were validated. We proceed now to extract some useful information from them.

4. 1. Mean and variance

The following table presents the mean and variance of the most important distributions in each aggregation strategy. These formulas were calculated with the help of the MGFs presented in section 2. Indeed [15], the mean of a distribution is the first derivate of its MGF with respect to s evaluated in s = 0. The variance is the difference between the second moment and the square of the mean. The second moment of a distribution is the second derivate of its MGF with respect to s evaluated in s = 0.

Table 1. Average and variance of the most important distributions. Int[.] represents the highest integer smaller than its argument.

Burst Aggregation Strategy	Timeouts	Buffer limit	Packet count	Random Selection
Average number of IP packets per burst E[N]	$n = \lambda * T + 1$	$n = Int\left[\frac{B_{size}}{E[S]}\right]$	n	n = 1 / p
Variance of the number of IP packets per burst Var[N]	n-1	Depends on S, see equation 6	0	n*(n-1)
Average burst formation time E[T]	$T = \frac{n-1}{\lambda}$	$\frac{n-1}{\lambda}$	$\frac{n-1}{\lambda}$	$\frac{n-1}{\lambda}$
Variance of the burst formation time Var[T]	0	Depends on S, see equation 7	$\frac{N-1}{\lambda^2}$	$\frac{N\cdot(N-1)}{\lambda^2}$
Mean interarrival time between bursts E[Z]	$T+\frac{1}{\lambda}=\frac{n}{\lambda}$	$\frac{n}{\lambda}$	$\frac{n}{\lambda}$	$\frac{n}{\lambda}$
Variance of the interarrival time between bursts Var[Z]	$\frac{1}{\lambda^2}$	$\frac{n+Var[N]}{\lambda^2}$	$\frac{n}{\lambda^2}$	$\frac{n^2}{\lambda^2}$
Average burst size E[B]	$n\cdot E[S]$	$n\cdot E[S]$	$n\cdot E[S]$	$n\cdot E[S]$
Variance of the burst size Var[B]	$n\cdot Var[S] +$ $+ (n-1)\cdot E^2[S]$	Depends on S, see equation 11	$n\cdot Var[S]$	$n\cdot Var[S]+n\cdot$ $\cdot(n-1)\cdot E^2[S]$

Observe that the average interarrival time between bursts, burst size and burst formation time are the same for all the aggregation strategies. What changes from one aggregation strategy to another is the variance (and higher moments) of the random variables.

4. 2. Burst distribution behavior with high traffic aggregation

Bursts grow bigger depending on the parameter of each aggregation strategy and on the traffic load conditions (see table 1 for details). For constant average IP packet size, the burst size depends only on the average number of IP packets per burst n. The impact of high traffic aggregation on the burst size distribution for the case of the aggregation strategy with timeouts and a tri-modal IP packet size distribution is shown in figure 8. The results were obtained from equation 4.

As n increases bursts grow bigger and the burst size distribution becomes smoother and tends to a Gaussian form. This is in accordance with the Central Limit Theorem and has also been noticed in [3]. In addition, figure 8 gives us an idea about the speed of convergence. Note that for n = 50 packets the Gaussian form can be clearly recognized despite the fact that in figure 8 we have started with a tri-modal distribution for the IP packet size, which does not resemble a Gaussian distribution at all. Similar results were obtained for the rest of the aggregation strategies presented in this work.

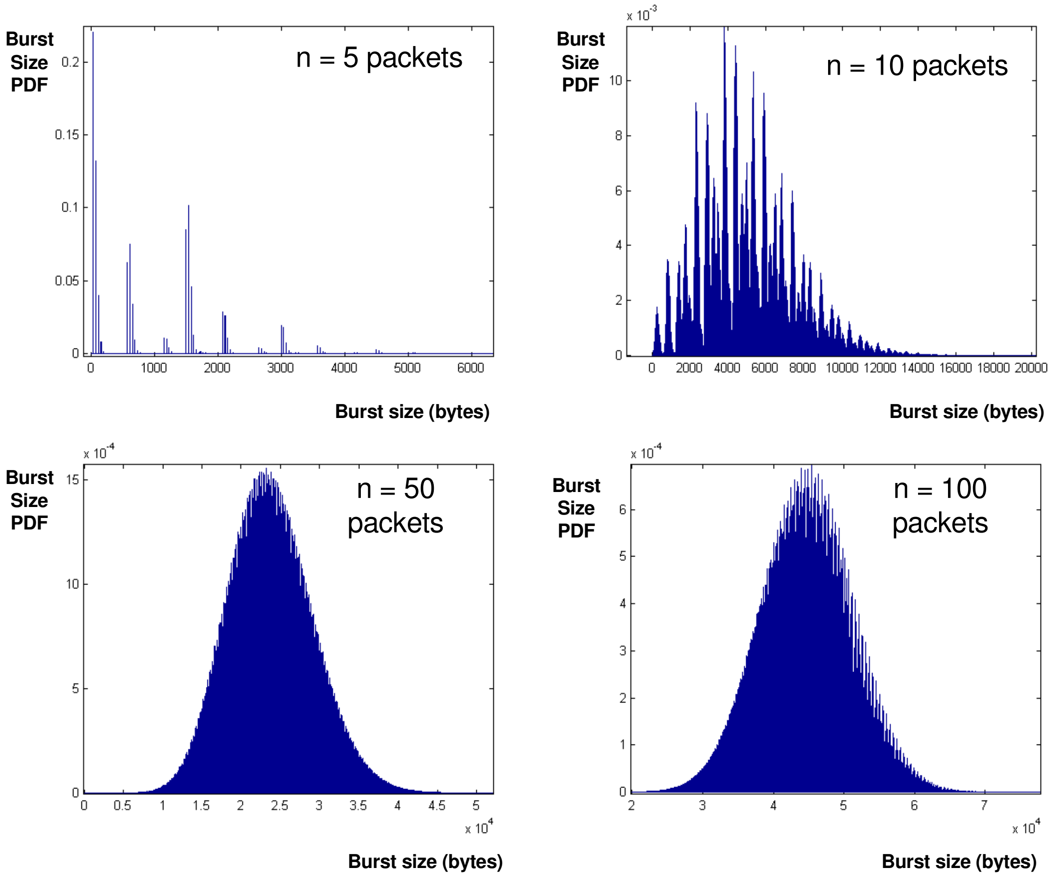


Figure 8: Burst size distribution for four different parameter configurations and the aggregation strategy of timeouts. The variable n represents the average number of IP packets per burst.

In most practical applications of OBS networks the average number of IP packets per burst will be fairly above 50 (just consider that according to measurements [14] 60 % of the IP packets have a size of only 40 bytes). We can conclude therefore that for most OBS networks the burst size distribution can be assumed to be Gaussian with mean and variances given in table 1.

4.3. Outline of the burst aggregation strategies

Table 2 outlines the most important characteristics of the different burst aggregation strategies presented.

Most of the distributions are well-known, but we see once more that the only aggregation strategy that leads to poisson traffic is the random selection strategy, since it is the only one that presents exponentially distributed interarrival times. Consequently, this is the only strategy for which an exact blocking probability formula is available (i.e. the Erlang B formula).

Table 2. Main probability distributions for the different aggregation strategies.

Burst Aggregation Strategy	Timeouts	Buffer limit	Packet count	Random Selection
Number of IP packets per burst N	Poisson	Depends on *S*, see equation 6	Constant	Geometric
Burst formation time T	Constant	Depends on *S*, see equation 7	Erlang	A function of Exponential and delta r.v., see equation 17
Interarrival time distribution between bursts Z	Shifted exponential	Depends on *S*, see equation 8	Erlang	Exponential
Burst size distribution B for N > 100	Normal	Depends on *S*, see equation 11	Normal	Exponential
Queuing model	G/G/..	G/G/..	G/G/..	M/G/..
Blocking Probability	No analytical model available	No analytical model available	No analytical model available	Erlang-B formula

5. Conclusions

We have presented the first complete and exact analytical study of some of the most important state-of-the-art aggregation strategies (timeouts, buffer limit and packet count). The study is complete in the sense that no simplification of the aggregation strategies has been made and exact in the sense that no approximation of the calculations has been used.

Many authors in the literature (see for instance [8][9][10]) have assumed that the burst traffic resulting from the aggregation of user packets can be accurately modeled with a Poisson process. With our analytical models we have demonstrated that for the three studied aggregation strategies the traffic at the burst level is definitely not Poisson. This implies that the Erlang B formula can not be used as an exact formula in order to calculate blocking probabilities in such an OBS network.

We have presented a new aggregation strategy the main feature of which is that it leads to a Poisson distributed burst arrival process, and therefore allows the use of the Erlang B formula and of the many useful results described in the literature which were based on the assumption of poisson traffic. A complete and exact analytical study of this new aggregation strategy was presented, as well as an easy way to implement it in the edge nodes requiring only software modifications.

In our analytical models for each aggregation strategy we provided tractable expressions for the distributions of the burst interarrival time, burst size, burst formation time and number of IP packets per burst. In addition in table 1 we presented the average and variance of the above mentioned distributions for each aggregation strategy. Moreover, for standard OBS networks it was shown that the burst size distribution can be approximated with a Gaussian distribution with mean and variance given in table 1.

Finally, the validation of our analytical models showed a perfect matching between the model predictions and the simulation results, which shows again the fact that our analytical models are complete and exact.

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