HighSpeed TCP Modeling and Analysis

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Abstract

Since the traditional TCP congestion control mechanism can not work efficiently in Gigabit high speed network, a variety of TCP, which is called HighSpeed TCP(HSTCP), is proposed. In this paper we develop a HSTCP model based on small time scale, and get the transfer function of HSTCP/RED closed-loop system through linearizing the HSTCP model. We then use two regular methods in control theory, namely gain margin and phase margin, to analyze the relative stability of HSTCP. The results indicate that HSTCP is more stable than TCP when link bandwidth, flow number and round-trip time change respectively. Finally we evaluate the fairness index of HSTCP/TCP hybrid system, and we find that the heavier the aggregate traffic difference between these two class flows, the higher the fairness of hybrid system.

1 Introduction

With the rapid development of Internet, high speed links such as optical fiber etc have been widely employed in practice in order to satisfy the imminent bandwidth requests of a large number of multimedia and real-time applications. Whereas some recent studies indicate that the traditional TCP can not make full use of the abundant bandwidth resource in Gigabit high speed network due to the slow response characteristic of TCP [1][2]. For example, supposing a connection has a round-trip time of 100ms and a packet size of 1500bytes, an available bandwidth of 10Gbps corresponds to a congestion window size of 83333 packets. According to the Additional Increase Multiplicative Decrease(AIMD) in TCP congestion control, after an immediate detection of a congestion event, the congestion window will be set to 41667 packets. The longest time interval that congestion avoidance phase would go through will be 4167 seconds, roughly 1.2 hours. Furthermore, from the response function with congestion window size and per-packet loss rate($W = 1.22 / p^{0.5}$ [3]), the congestion window size can maintain 83333 packets only if the packet loss rate is to be 2×10^{-10}. It means that only one packet permits to be dropped among 5×10^{9} packets, namely, only one drop event occurs per 1.7 hours. Obviously this packet loss rate request is unrealistic in actual Internet.

In order to solve these problems, researchers have proposed several novel transmission protocol, i.e. HighSpeed TCP[1], Scalable TCP(STCP)[2]， Fast AQM Scalable TCP(FAST TCP)[4]，Explicit Congestion Control Protocol(XCP)[5], etc. HSTCP and STCP use packet loss event to detect congestion, just similar to traditional TCP. The major improvement of those methods is to alter the AIMD strategy, which make the congestion window size increase more aggressively in congestion avoidance phase, and decrease more alleviatively after a packet drop has been detected. FAST TCP is similar to TCP Vegas, which uses the queuing delay to predict congestion, and utilizes a smooth window adjustment mechanism to update the congestion window size. XCP is a novel transmission protocol undertaken by both end system and router. In the packets transmission processing, the router notifies the end systems the available bandwidth via some special options in packet header and XCP aims to reduce the congested possibility as maximum as possible. In these protocols, the advantages of HSTCP are that it offers flexibility, and effectively scale to wide range of available bandwidths. It is a simple and straightforward extension adaptive altering to the existing congestion control mechanism, which can be implemented and deployed in Internet fleetly.

2 Motivation

At present there are three main approaches to evaluate HSTCP, namely, (1) Performance analysis based on measurement via Testbed [6]. (2) Performance analysis based on network simulation via ns-2 [7]. (3) Performance analysis based on theoretical model. Although the first and second approachs are accurate to evaluate the network, their limitations are very clear. For example, they can not describe the relationships between network configurable factors explicitly [8]. In addition, they will take a long time to program and run a test script about Gigabit high speed network, and need lots of storage spaces to support precise data statistics. Thus in this paper we use the third approach, which can describle the relationships between network configurable factors explicitly, and take only a short time and a little space to get the approximate results.

Although some existing studies find that HSTCP can achieve much higher utilization and throughput than traditional TCP, theoretically analyses about stability and fairness have seldom been found. The contribution of this paper is that we apply stochastic process to develop a HSTCP model based on small time scale, and utilize two regular performance standards in traditional control theory, gain margin(GM) and phase margin(PM), to analyz and compare the relative stability of HSTCP/RED closed-loop system and TCP/RED system. We also investigate the fairness index of HSTCP/TCP hybrid system in which the high bandwidth resource is shared by HSTCP and TCP flows.

The rest paper is structured as follows. Section 3 introduces the HSTCP congestion control mechanism, and describes how to model and linearize the HSTCP model detailed. Section 4 presents the influences of bandwidth, flow number and round-trip time on relative stability respectively, as well as evaluation of the fairness index of HSTCP/TCP hybrid system based on the model. Section 5 summarizes what has been achieved in this paper and gives directions for future works.

3 HSTCP Modeling

3.1 HSTCP Congestion Control Mechnaism

The AIMD(a,b) congestion control mechanism in traditional TCP is given by:

$$ACK: \quad W \leftarrow W + \frac{a}{W}$$
$$DROP: \quad W \leftarrow (1-b)W$$

That is, after a packet is dropped, the congestion window size decreases from W to $(1-b)W$, and otherwise the congestion window size increases from W to $W+a$ each round-trip time.

HSTCP updates the congestion control mechanism as follows:

$$ACK: \quad W \leftarrow W + \frac{a(W)}{W}$$
$$DROP: \quad W \leftarrow (1-b(W))W$$

In other words, HSTCP let the additional and multiplicative parameters $a(W)$ and $b(W)$ be functions of the current congestion window size. When the congestion window size changes, these two parameters will adjust accordingly. At steady state, TCP's AIMD(a,b) response function is given by [9]:

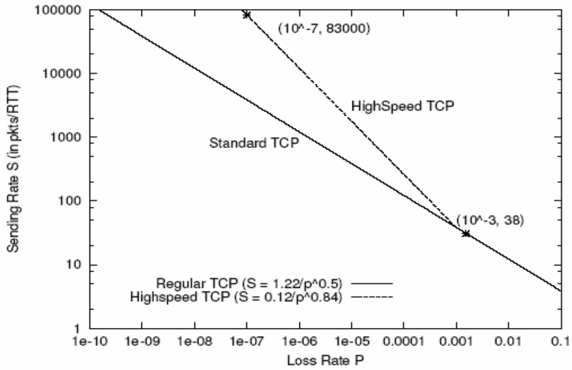

Figure 1. HSTCP and TCP response functions [9]

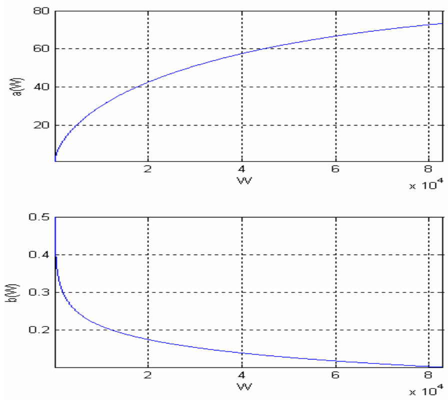

Figure 2. *a(W)* and *b(W)*

$$W = \sqrt{\frac{a \cdot (2-b)}{2bp}}$$

where p is the per-packet drop rate. HSTCP also chooses the additional and multiplicative parameters $a(W)$ and $b(W)$ to satisfy the following equation:

$$W = \sqrt{\frac{a(W) \cdot (2-b(W))}{2b(W)\,p}} \tag{1}$$

Since HSTCP have to achieve a high speed bandwidth with realistic packet loss rate, Sally Floyd proposed a novel response function in [1]. Figure 1 shows the response function of HSTCP and TCP respectively. Given the two fixed points (P_L, W_L) and (P_H, W_H) for response function of HSTCP, Sally Floyd assumes that the new response function is linear on a log-log scale, so the response function of HSTCP can be write as:

$$W = p^S (1/P_L)^S W_L \tag{2}$$

where S is the slope of the HSTCP response line,

$$S = \frac{\log W_H - \log W_L}{\log P_H - \log P_L} \tag{3}$$

Taking account of the compatibility with the standard TCP and the general error rate of high speed optical fiber,

(P_L, W_L) and (P_H, W_H) are fixed at (10^{-3}, 38) and (10^{-7}, 83000). Substituting these values into equation (2), we have

$$W = \frac{0.12}{p^{0.83}} \tag{4}$$

From equation (1) and (4), the increase parameter $a(W)$ can be simplify to

$$a(W) = 0.16 \cdot W^{0.8} \cdot \frac{b(W)}{2 - b(W)} \tag{5}$$

As for the decrease parameter $b(W)$, let it vary linearly as the log of W, and restrict it between 0.5 and 0.1 when W vary from 38 packets to 83333 packets. Thus the decrease parameter equation is expressed as:

$$b(W) = -0.12 \log W + 0.69 \tag{6}$$

Figure 2 shows the curves of $a(W)$ and $b(W)$ when W varying from 38 to 83333.

Consequently, the congestion control mechanism of HSTCP can be described with equation (4), (5) and (6). Through the novel response function (4) which is suitable for high speed network with realistic packet loss rate, the author inversely deduces the relationship equations between the increase parameter and decrease parameter and current congestion window.

3.2 HSTCP Model

Before construct the model of HSTCP, we make some reasonable assumptions following existing models [10], [11] , [12], [13]:

1) The HSTCP connection is a long-term connection, and it always operates in the congestion avoidance phase at steady state.
2) The buffer is so large enough that it will never overflow, and RED algorithm which is the most regular AQM policy is employed in this model.
3) Packets are dropped independently, and at most one packet is dropped each round-trip time.
4) Omitting processing delay of all packets and queuing delay of acknowledge packets, define the round-trip time R as round-trip propagating delay T_p plus queuing delay of data packet q/C ($R = T_p + q/C$), where q is the queue length, C is the link bandwidth.

We take a round-trip time interval as the time scale in this model. Let the size of current congestion window and the per-packet drop rate be denoted as W and p respectively. If the sender can detect the drop event via the ith acknowledge packet, the probability of this event is $(1-p)^{W-1} \cdot p$ under assumption (3), and the current congestion window size is $W + i \cdot a(W)/W$, after one round-trip time, the total change in the congestion window size is $a(W) - b(W)(W + i \cdot a(W)/W)$. Otherwise if no packets are dropped in a round-trip time, the corresponding probability is $(1-p)^W$, and after one round-trip time the total change in the congestion window size is $a(W)$. Consequently, the expected change in the congestion window size in a round-trip time can be expressed as:

$$\begin{aligned}\overline{\Delta W} &= (1-p)^W \cdot a(W) + \sum_{i=1}^{W} (1-p)^{W-1} p \cdot \left[a(W) - b(W) \cdot (W + \frac{a(W)}{W} \cdot i)\right] \\ &= (1-p)^W \cdot a(W) + (1-p)^{W-1} p \cdot \left[W \cdot a(W) - W^2 b(W) - a(W) \cdot b(W) \cdot \frac{W+1}{2}\right]\end{aligned} \tag{7}$$

when p is small, $W >> 1$, $(1-p)^W \approx 1 - pW$, we keep first-order items of p while omitting the high-order items of p, and equation (7) will be simplified to:

$$\begin{aligned}\overline{\Delta W} &\approx (1 - pW) \cdot a(W) + (1 + p - pW) p \\ &\quad \cdot \left[W \cdot a(W) - W^2 b(W) - a(W) \cdot b(W) \cdot \frac{W+1}{2}\right] \\ &= a(W) - b(W) W^2 \cdot p - a(W) \cdot b(W) \cdot \frac{W+1}{2} \cdot p + o(p) \\ &\approx a(W) - b(W) W^2 \cdot p - a(W) \cdot b(W) \cdot \frac{W}{2} \cdot p\end{aligned} \tag{8}$$

Since $dW/dt = \overline{\Delta W}/R$, we have:

$$\dot{W} = \frac{a(W)}{R} - \frac{b(W) W^2}{R} \cdot p - \frac{a(W) \cdot b(W) \cdot W}{2R} \cdot p \tag{9}$$

Given there is approximately one round-trip time delay between the time when a drop event occurs at the queue and the time when the drop notification reaches the sender, i.e. $p \approx p(t-R)$, we can update the equation (9) as:

$$\dot{W} = \frac{a(W)}{R} - \frac{b(W) W^2}{R} \cdot p(t-R) - \frac{a(W) \cdot b(W) \cdot W}{2R} \cdot p(t-R) \tag{10}$$

Substituting $a(W) = 1$ and $b(W) = 1/2$ into equation (10), we reach the differential equation of traditional TCP:

$$\dot{W} = \frac{1}{R} - \frac{W^2}{2R} \cdot p(t-R) - \frac{W}{4R} \cdot p(t-R) \tag{11}$$

Assuming there are N homogeneous HSTCP flows going through a bottleneck link, and the queue will never drain, the equation describing the behavior of q is given by:

$$\dot{q} \approx -C + N \cdot \frac{W}{R} \tag{12}$$

The first item corresponds to the decrease in the queue length, and the second item models the increase in the queue in the queue length due to the arrival of packets from HSTCP flows.

In summary, the model in which N homogeneous HSTCP connections traverse a bottleneck link can be approximated by the following differential equations:

$$\begin{cases} \dot{W} = f(W, p, q) = \dfrac{a(W)}{R} - \dfrac{b(W) W^2}{R} \cdot p(t-R) \\ \qquad\qquad - \dfrac{a(W) \cdot b(W) \cdot W}{2R} \cdot p(t-R) \\ \dot{q} = g(W, q) \approx -C + N \dfrac{W}{R} \end{cases} \tag{13}$$

where

$$R = T_p + \frac{q}{C}$$

The relationship between p and q is determined by the AQM policy. In the rest of the paper, we will use the regular RED algorithm to analyze the performance of HSTCP/RED closed-loop system.

3.3 Model Linearization

Since the HSTCP model (equation 13) is nonlinear, it is impossible to analyze it through the maturem and well-known linearization system theory. We will linearize this model in small-deviation around the operation point. The operation point (W_0, q_0, p_0) is defined by $\dot{W}\big|_{W=W_0} = 0$ and $\dot{q}\big|_{q=q_0} = 0$ so that

$$\begin{cases} \dot{W}\big|_{W=W_0} = 0 \quad \Rightarrow \quad p_0 = \dfrac{2a(W_0)}{b(W_0)\cdot W_0 \cdot (2W_0 + a(W_0))} \\ \dot{q}\big|_{q=q_0} = 0 \quad \Rightarrow \quad W_0 = \dfrac{RC}{N} \end{cases} \tag{14}$$

where the small-deviation around the operation point can be expressed by:

$$\begin{cases} \delta W = W - W_0 \\ \delta q = q - q_0 \\ \delta p = p - p_0 \end{cases} \tag{15}$$

We linearize equation (13) about the operating point as following:

$$\begin{cases} \delta \dot{W} = -K_1 \cdot \delta W - K_2 \cdot \delta p \cdot e^{-sR} \\ \delta \dot{q} = K_3 \cdot \delta W - K_4 \cdot \delta q \end{cases} \tag{16}$$

where,

$$\begin{cases} K_1 \approx \dfrac{1.25 \cdot a(W_0)}{W_0 R} \\ K_2 = \dfrac{b(W_0)\cdot W_0 \cdot (2W_0 + a(W_0))}{2R} \\ K_3 = \dfrac{N}{R} \\ K_4 = \dfrac{1}{R} \end{cases} \tag{17}$$

In [11], the transfer function model for RED algorithm is:

$$C(s) = \frac{L_{red} \cdot K_{red}}{s + K_{red}} \tag{18}$$

where L_{red} is determined by the configurable parameters, i.e. drop probability, maximum threshold and minimum threshold, and K_{red} is determined by the queue averaging parameter and the sample time(see [10] for detailed).

In summary, the open-loop transfer function of whole HSTCP/RED control system is:

$$L(s) = \frac{L_{red} \cdot K_{red}}{s + K_{red}} \cdot \frac{K_2}{s + K_1} \cdot \frac{K_3}{s + K_4} \cdot e^{-sR} \tag{19}$$

Figure 3 shows the block diagram of the linearized HSTCP/RED closed-loop control system. Consider the frequency response function of the transfer function as:

$$L(j\omega) = \frac{L_{red} \cdot K_{red}}{j\omega + K_{red}} \cdot \frac{K_2}{j\omega + K_1} \cdot \frac{K_3}{j\omega + K_4} \cdot e^{-j\omega R} \tag{20}$$

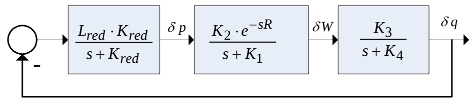

Figure 3. block diagram of the linearized HSTCP/RED closed-loop control system

4 Model Analysis

4.1 Relative Stability Analysis

In general, we are interested in not only the absolute stability of a system, but also how stable it is. The latter is often called relative stability [14]. In the frequency domain, Gain Margin (GM) and Phase Margin (PM) are always used to indicate relative stability. Gain margin is the amount of gain in decibels (dB) that can be added to the loop before the closed-loop system becomes unstable. Phase margin is defined as the angle in degrees through which the $L(j\omega)$ plot must be rotated about the origin so that the gain crossover passed through the (-1, j0) point. If the GM>0 and PM>0, the system is stable. Otherwise the system is unstable. Furthermore the greater the GM and PM value, the more stable the system.

This section is aimed to analyze relative stability of HSTCP. From proposition 1 in [11] (Appendix A), we know that if K_{red} and L_{red} is small, it is easy to meet stability criterion of TCP/RED closed-loop control system. Thus to reduce the influence on relative stability production by configurable parameters of RED algorithm, we let $K_{red} = 10^{-4}$ and $L_{red} = 10^{-7}$ in the rest paper which are both small.

A. Link Bandwidth Influence on Relative Stability

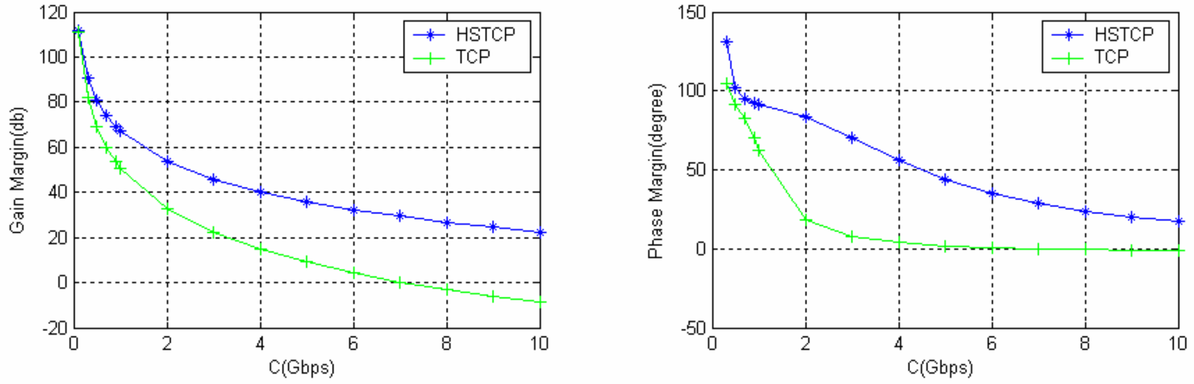

Figure 4. GM and PM vary with link bandwidth, where RTT=100ms, N=10.

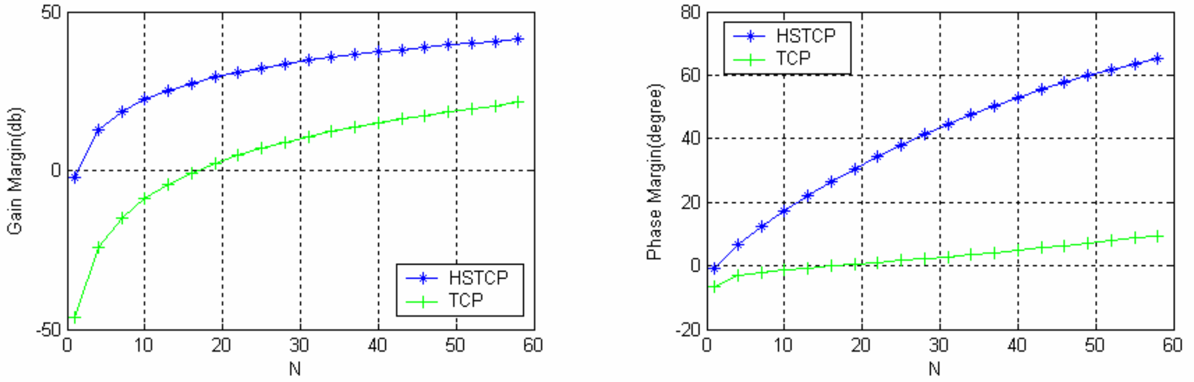

Figure 5. GM and PM vary with flow number, where RTT=100ms, C＝10Gbps.

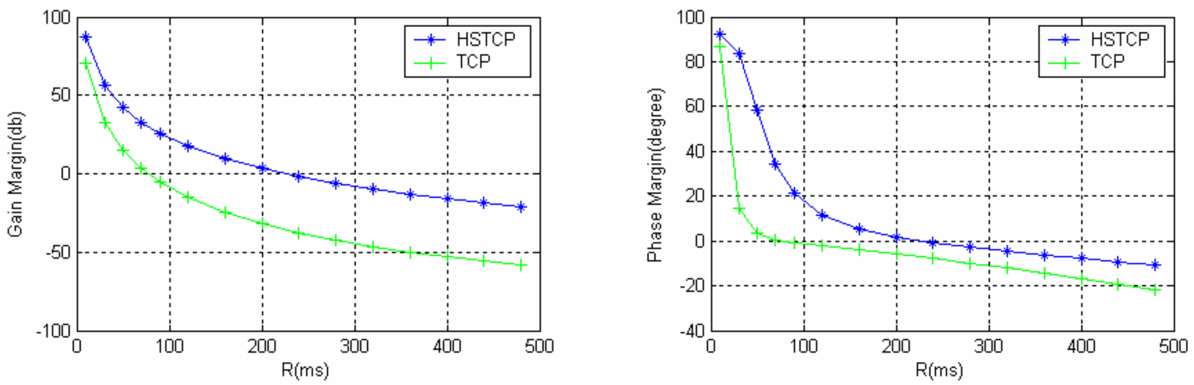

Figure 6. GM and PM vary with round-trip time, where N=10，C＝10Gbps

We first consider the influence on relative stability of link bandwidth. In this case, we suppose the link bandwidth *C* increase from 100Mbsp to 10Gbps which is completely shared by 10 individual homogeneous HSTCP flows, and the round-trip time is still kept at 100ms at steady state. The GM and PM diagram of HSTCP is plotted in figure 4. We also plot the GM and PM diagram of TCP in figure 4 to compare with HSTCP. The behavior of TCP is modeled by equation (14), and its linearized process is similar to the process of HSTCP (the detailed steps is ignored in this paper). As we can see, when link bandwidth increases gradually, GM and PM value of HSTCP and TCP are both strictly monotonic decrease. It indicates that the greater link bandwidth, the less stable HSTCP and TCP. Furthermore, we notice that GM and PM value of HSTCP always stay higher than those of TCP. This observation points outthat HSTCP's relative stability is higher than that of TCP.

B. Flow Number Influence on Relative Stability

Now we repeat the similar experiment in which the link bandwidth is fixed to 10Gbps. Figure 5 shows that the GM and PM diagram of HSTCP and TCP when flow number *N* vary from 1 to 60. As we observe, increasing the value of flow number results in increasing GM and PM value of HSTCP and TCP, that is to say, the more flows going through the link, the more stable HSTCP and TCP. Notice that the changes in HSTCP PM value is greater than the changes in TCP PM value when N is increasing gradually, it indicates that the gap in stability between HSTCP and TCP is increasing with flow number increasing. Finally note that HSTCP goes into steady state if the flow number greater than 2, however TCP does not go into steady state till flow number greater than 15. We can conclude that given the link bandwidth of system, a small HSTCP flow can make system stable, but much more TCP flows are needed to achieve the same target.

C. Round-Trip Time Influence on Relative Stability

In this case, we let $N = 10$ and $C = 10Gbps$, and figure 6 shows that the GM and PM diagram of HSTCP and TCP when round-trip time vary from 10ms to 500ms. Clearly, no matter how round-trip time changes, GM and PM value of HSTCP are always higher than GM and PM value of HSTCP.

It's important to note how to choose the configurable parameters of RED algorithm. This paper does not discuss the effects of RED's configuration parameters on the relative stability of a HSTCP/RED closed-loop sytem. In fact the detailed discussing of RED parameters is represented in [10] and [11]. Thus we do not repeat this issue in our paper. We also find that if we change the configurable parameters of RED algorithm, the relative stability synchronously changes, but the stability gap between HSTCP and TCP still exists. The relative stability of HSTCP is always higher than the relative stability of TCP when configurable parameters of RED algorithm changes.

4.2 Fairness index Analysis

In [15], a formula used to evaluate congestion control mechanism is proposed by Raj Jain:

$$F = \frac{(\sum n x_i)^2}{n \sum x_i^2} \tag{20}$$

where n corresponds to the flow number in network, x_i is the throughput of the *i*th flow. If all x_i is equal, thus *F*=1, the fairness of hybrid system will be the highest. Based on equation (20) we will show the fairness index of HSTCP/TCP hybrid system quantificationally.

Suppose there are N_1 homogeneous HSTCP flows and N_2 homogeneous TCP flows going through the bottleneck

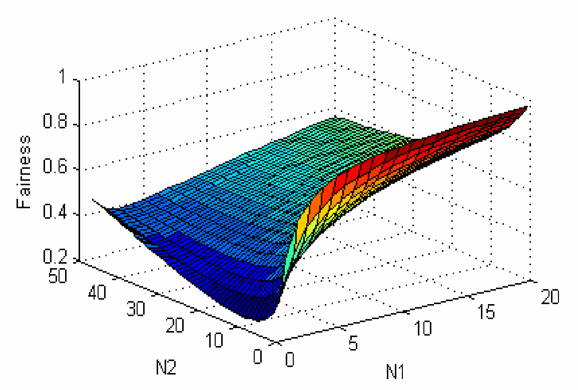

Figure 7. Fairness index varies with N_1 and N_2

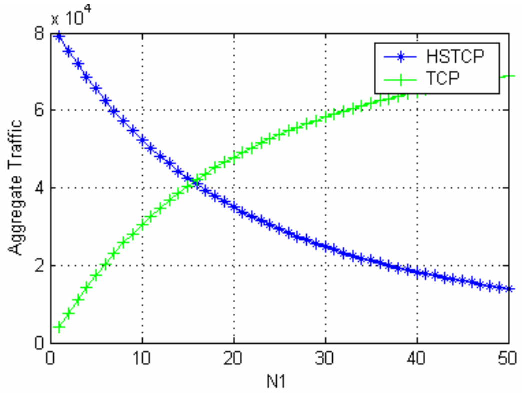

Figure 8. Aggregate Traffic varies with N_2 where N_1=1.

link, the round-trip time of them are all equal, and the congestion window at steady state are denoted by W_1 and W_2, respectively. Through HSTCP model (equation (13)), when the hybrid system goes into steady state, the following equation is satisfied:

$$\frac{N_1 W_1}{R} + \frac{N_2 W_2}{R} = C \tag{23}$$

Where,

$$\begin{cases} W_1 = 0.12 / p^{0.83} \\ W_2 = 1.22 / p^{0.5} \end{cases} \tag{24}$$

So the fairness index of HSTCP/TCP hybrid system can be written as:

$$F = \frac{(N_1 \cdot \frac{W_1}{R} + N_2 \cdot \frac{W_2}{R})^2}{(N_1 + N_2) \cdot (N_1 \cdot \frac{W_1^2}{R^2} + N_2 \cdot \frac{W_2^2}{R^2})} = \frac{C^2}{C^2 + N_1 N_2 (W_1 - W_2)^2} \tag{25}$$

It is clear that, when the link bandwidth and round trip time is fixed, the fairness is determined by N_1 and N_2.

Let link bandwidth be 10Gbps, and round-trip time still be 100ms. Figure 7 shows the fairness of hybrid system varies with N_1 and N_2 in this situation. Figure 8 shows the corresponding aggregate traffic of HSTCP and TCP respectively according to fairness of hybrid system varies with N_2 where N_1 is fixed to be 1. Synthesizing these figures, we can conclude that the heavier the aggregate traffic difference between these two class flows, the higher the fairness index of hybrid system.

5 Conclusion and Future Work

We develop a HSTCP fluid-flow model based on small time scale, and analyze the stability of HSTCP/RED closed-loop system and the fairness of HSTCP/TCP hybrid system. The modeling method and relative stability analytical method in this paper are novel and useful for investigating high speed transmission domain. It can be used to analyze the stability of transmission protocol and tune the AQM configurable parameters. In future research we will apply it to analyze HSTCP, Scalable TCP, FAST and XCP further, and even assist in designing a novel high speed transmission protocol or an AQM algorithm.

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Appendix A.

Proposition 1: Let K_{red} and L_{red} satisfy

$$\frac{L_{red}(R^+C)^3}{(2N^-)} \le \sqrt{\frac{\omega_g^2}{K^2}+1},$$

where

$$\omega_g = 0.1\min\{\frac{2N^-}{(R^+)^2C}, \frac{1}{R^+}\}$$

Then, the linear feedback control system using $C(s)=C_{red}(s)$ is table for all $N \ge N^-$ and all $R_0 \le R^+$.

This proposition is the stability criterion of TCP/RED closed-loop system. The inequality term note that if the smaller of K_{red} and L_{red} value, the easier the criterion to be satisfied.